ព្រះរាជាណាចក្រកម្ពុជា

លីមីត - អនុគមន៍

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

សំរាប់ប្រលង

មហាវិទ្យាល័យ / វិទ្យាស្ថាន

ប.ធ

[illegible]

អារម្ភកថា

សៀវភៅ "សិក្សាអនុគមន៍ ថ្នាក់ទី ១១" នេះ អាចចាត់ទុកជាស្នាដៃទី ១ របស់អ្នករៀបរៀងដែលប្រមូលចេញពីឯកសារជាច្រើន ដើម្បីជាជំនួយក្នុងការសិក្សារបស់ប្រិយមិត្តនិងប្អូនៗ ។

ក្នុងសៀវភៅនេះ យើងមានសង្ខេបមេរៀន រូបមន្តសំខាន់ៗ ព្រមនិងដំណោះស្រាយនៃលំហាត់ ។ នៅចុងបញ្ចប់នៃសៀវភៅ យើងបានរៀបចំលំហាត់សាកល្បងវិញ្ញាសាមួយចំនួន ដែលភាគច្រើនជាលំហាត់ពិបាកៗ ហើយដែលអាចប្រកួតពិន្ទុឡើងវិញ្ញាសានេះ ដើម្បីជាមូលដ្ឋានរបស់ប្អូនៗ ។

ដោយបទពិសោធន៍នៅមានកំរិតដូចនេះ ពិតជាមានកង្វះខាតខ្លះៗពុំខាន ។ យើងខ្ញុំរង់ចាំទទួលការរិះគន់ ស្ថាបនាពីប្រិយមិត្តដោយរីករាយ ដើម្បីកែលំអសៀវភៅនេះ ពេលក្រោយកាន់តែល្អប្រសើរថែមមួយកំរិតទៀត ។

សូមប្រិយមិត្តទទួលបានជោគជ័យក្នុងការសិក្សា

ប. ជ

ទ្រឹស្តីបទនិងវិធានគណនាលីមីតខ្លះៗ

យើងបានដឹងរួចមកហើយថា លីមីតគឺជាផ្នែកសំខាន់មួយនៃវិភាគ (វិភាគគណិត) ព្រោះជាជំនួយនិងសិក្សាភាពជាប់, ដេរីវេ (ដេរីវេនិយមន័យ) ; អាស៊ីមតូត គណនាអាំងតេក្រាលកំនត់ (និយមន័យអាំងតេក្រាលកំនត់)... គេត្រូវតែប្រើលីមីត ។ ក្នុងនេះ យើងគួរតែដោះគណនាលីមីត មានរាងមិនកំនត់ទាំងឡាយ នៃលីមីត ដែលយើងតែងជួបមាន 7 គឺ

$\frac{0}{0}$; $\frac{\infty}{\infty}$; $0\times\infty$; $\infty-\infty$; 1^{∞} ; ∞^{0} ; 0^{0}

ឥឡូវនេះ យើងសូមលើកយកវិធានគណនា, ឧទាហរណ៍ និង ទ្រឹស្តីបទខ្លះៗ មកបង្ហាញ

1. វិធានគណនាលីមីតរាងមិនកំនត់ $\frac{0}{0}$

ឧបមាថា $\lim\limits_{x\to a}\frac{f(x)}{g(x)}$ មានរាងមិនកំនត់ $\frac{0}{0}$

ដើម្បីគណនាលីមីតប្រភេទនេះ គេត្រូវ

- បំបែក $f(x)$ និង $g(x)$ អោយមានកត្តា $(x-a)$; (ឬ $(x-a)^2$ ...)
- សំរួលកត្តា $(x-a)$ ដោយដើម្បីបំបាត់រាងមិនកំនត់ (ព្រោះកាលណា $x\to a$ នោះ $x\neq a$ ឬ $x-a\neq 0$ ក្នុងនេះគេអាចសំរួលកត្តា $(x-a)$ បាន)
- ជំនួសតំលៃ x ដោយ a ទៅក្នុងលីមីត

ម្យ៉ាងទៀត ដើម្បីងាយការបំបែក $f(x)$ និង $g(x)$ ជាផលគុណកត្តាមានភាពងាយស្រួល យើងសូមរំលឹកនូវលក្ខណភាពសំខាន់ៗ មួយចំនួន

$$a^2-b^2=(a-b)(a+b)$$

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$a^n-b^n=(a-b)(a^{n-1}+a^{n-2}b+a^{n-3}b^2+\dots+ab^{n-2}+b^{n-1});\forall n$$

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

$$a^5+b^5=(a+b)(a^4-a^3b+a^2b^2-ab^3+b^4)$$

$$a^n+b^n=(a+b)(a^{n-1}-a^{n-2}b+\dots+(-1)^p a^p b^{n-p}+\dots+b^{n-1}),\ n \text{ ជាចំនួនគត់សេស}$$

ឧទាហរណ៍ : គណនា $\lim\limits_{x\to -1}\frac{x^{2n+1}+1}{x^{2m+1}+1}$

កាលណា $x\to -1$ លីមីតនេះមានរាងមិនកំនត់ $\frac{0}{0}$

យើងបាន :

$$\lim_{x\to -1}\frac{x^{2n+1}+1}{x^{2m+1}+1}=\lim_{x\to -1}\frac{(x+1)(x^{2n}-x^{2n-1}+\dots-x+1)}{(x+1)(x^{2m}-x^{2m-1}+\dots-x+1)}$$

$$=\lim_{x\to -1}\frac{x^{2n}-x^{2n-1}+\dots-x+1}{x^{2m}-x^{2m-1}+\dots-x+1}=\frac{\overbrace{1+1+\dots+1}^{(2n+1)\text{ តួ}}}{\underbrace{1+1+\dots+1}_{(2m+1)\text{ តួ}}}=\frac{2n+1}{2m+1}$$

ដូចនេះ $\lim\limits_{x \to -1} \dfrac{x^{2n+1}+1}{x^{2m+1}+1} = \dfrac{2n+1}{2m+1}$

ឧទាហរណ៍៖ គណនា $\lim\limits_{x \to 1} \dfrac{\sqrt[n]{x}-1}{\sqrt[m]{x}-1}$

កាលណា $x \to 1$ លីមីតនេះមានរាងមិនកំណត់ $\frac{0}{0}$

យើងតាង $x = y^{mn}$; កាលណា $x \to 1$ នោះ $y \to 1$

យើងបាន៖

$$\lim_{x \to 1} \frac{\sqrt[n]{x}-1}{\sqrt[m]{x}-1} = \lim_{y \to 1} \frac{\sqrt[n]{y^{mn}}-1}{\sqrt[m]{y^{mn}}-1} = \lim_{y \to 1} \frac{y^m-1}{y^n-1}$$

$$= \lim_{y \to 1} \frac{(y-1)(y^{m-1}+y^{m-2}+\dots+y+1)}{(y-1)(y^{n-1}+y^{n-2}+\dots+y+1)}$$

$$= \lim_{y \to 1} \frac{y^{m-1}+y^{m-2}+\dots+y+1}{y^{n-1}+y^{n-2}+\dots+y+1} = \frac{\overbrace{1+1+\dots+1}^{m \text{ តួ}}}{\underbrace{1+1+\dots+1}_{n \text{ តួ}}} = \frac{m}{n}$$

ដូចនេះ $\lim\limits_{x \to 1} \dfrac{\sqrt[n]{x}-1}{\sqrt[m]{x}-1} = \dfrac{m}{n}$

---

2 វិធានគណនាលីមីតរាងមិនកំណត់ $\frac{\infty}{\infty}$

ចំពោះលីមីត $\lim\limits_{x \to \infty} \dfrac{f(x)}{g(x)}$ មានរាងមិនកំណត់ $\frac{\infty}{\infty}$

ដើម្បីគណនាលីមីតប្រភេទនេះ គេត្រូវ

- ដាក់តួមេគុណ x ដែលមានដឺក្រេខ្ពស់ជាងគេក្នុង $f(x)$ និង $g(x)$ ជាកត្តារួម
- សម្រួលកត្តានោះចោល (ព្រោះ $x \to \infty$ នោះ $x \neq 0$)
- គេឱ្យ $x \to \infty$ យើងនឹងបានលទ្ធផល

* ចំណាំ៖ ពហុធា $P(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a^{n-1}x + a^n$

កាលណា $x \to \infty$ នោះ $P(x) \simeq a_n x^n$

ឧទាហរណ៍៖ គណនា $\lim\limits_{x \to \infty} \dfrac{3x^2-7}{\sqrt{x^4-3x+4}}$

លីមីតនេះមានរាងមិនកំណត់ $\frac{\infty}{\infty}$ កាលណា $x \to \infty$

យើងបាន៖

$$\lim_{x \to \infty} \frac{3x^2-7}{\sqrt{x^4-3x+4}} = \lim_{x \to \infty} \frac{x^2\left(3-\frac{7}{x^2}\right)}{x^2\sqrt{1-\frac{3}{x^3}+\frac{4}{x^4}}} = \lim_{x \to \infty} \frac{3-\frac{7}{x^2}}{\sqrt{1-\frac{3}{x^3}+\frac{4}{x^4}}} = 3$$

ដូចនេះ

$$\lim_{x \to \infty} \frac{3x^2-7}{\sqrt{x^4-3x+4}} = 3$$

ឧទាហរណ៍៖ គណនា $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}}{x+\sqrt{x^2+1}}$

លីមីតនេះមានរាងមិនកំណត់ $\frac{\infty}{\infty}$ កាលណា $x\to+\infty$

ដំណោះស្រាយ៖

$$\lim_{x\to+\infty}\frac{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}}{x+\sqrt{x^2+1}}=\lim_{x\to+\infty}\frac{|x|\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+|x|\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}}{x+|x|\sqrt{1+\frac{1}{x^2}}}$$

$$=\lim_{x\to+\infty}\frac{x\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+x\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}}{x+x\sqrt{1+\frac{1}{x^2}}}$$ (ព្រោះ៖ $x\to+\infty$ នោះ $|x|=x$)

$$=\lim_{x\to+\infty}\frac{x\left(\sqrt{1+\frac{1}{x}+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x}+\frac{1}{x^2}}\right)}{x\left(1+\sqrt{1+\frac{1}{x^2}}\right)}=\frac{1+1}{1+1}=\frac{2}{2}=1$$

ដូចនេះ៖

$$\lim_{x\to+\infty}\frac{\sqrt{x^2+x+1}+\sqrt{x^2-x+1}}{x+\sqrt{x^2+1}}=1$$

3. វិធានគណនាលីមីតរាងមិនកំណត់ $0\times\infty$

កាលណា $x\to a$; $f(x)\to 0$; $g(x)\to\infty$ នោះ $\lim\limits_{x\to a} f(x)\cdot g(x)$ មានរាងមិនកំណត់ $0\times\infty$ ។ ដើម្បីគណនាលីមីតប្រភេទនេះ យើងបំលែងលីមីតនោះ មានរាងមិនកំណត់ $\frac{0}{0}$ ឬ $\frac{\infty}{\infty}$ ដោយសរសេរ $f(x)\cdot g(x)$ ជារាង

+ $f(x)\cdot g(x)=\dfrac{f(x)}{\frac{1}{g(x)}}$ ឬ $f(x)\cdot g(x)=\dfrac{g(x)}{\frac{1}{f(x)}}$

(រាងមិនកំណត់ $\frac{0}{0}$ ឬ $\frac{\infty}{\infty}$)

+ អនុវត្តន៍ទ្រឹស្តីបទ L'hospital : $\lim\limits_{x\to a}\dfrac{f_1(x)}{f_2(x)}=\lim\limits_{x\to a}\dfrac{f_1'(x)}{f_2'(x)}$

+ ត្រូវចាំថា ទ្រឹស្តីបទ L'hospital អនុវត្តន៍បានតែចំពោះលីមីតដែលមានរាងមិនកំណត់ $\frac{0}{0}$ ឬ $\frac{\infty}{\infty}$ ប៉ុណ្ណោះ ។

ឧទាហរណ៍៖ គណនា $\lim\limits_{x\to0}(x.\ln x)$

កាលណា $x\to0$ លីមីតមានរាងមិនកំណត់ $0\times\infty$

ដំណោះស្រាយ៖

$$\lim_{x\to0}(x.\ln x)=\lim_{x\to0}\frac{\ln x}{\frac{1}{x}}=\lim_{x\to0}\frac{(\ln x)'}{\left(\frac{1}{x}\right)'}$$

$$=\lim_{x\to0}\frac{\frac{1}{x}}{-\frac{1}{x^2}}=-\lim_{x\to0}x=0$$

ដូចនេះ៖ $\lim\limits_{x\to0}(x.\ln x)=0$

4. វិធានគណនាលីមីតរាងមិនកំណត់ $\infty-\infty$

គេជួបប្រទះរាងមិនកំណត់ $\infty-\infty$ នៅពេល $x\to\pm\infty$ ចំពោះកន្សោមដែលត្រូវគណនាលីមីតមានរាងឫស នោះគេប្រើកន្សោមឆ្លាស់ ដើម្បីបំបាត់រាងមិនកំណត់ $\infty-\infty$ នេះ ។ និងកត្តាសាធារណ៍ដែលគេនិយមប្រើ

គេមាន: $a^2-b^2=(a-b)(a+b)$

$a^3-b^3=(a-b)(a^2+ab+b^2)$

$a^n-b^n=(a-b)(a^{n-1}+a^{n-2}b+\cdots+ab^{n-2}+b^{n-1})$

ឧទាហរណ៍ គណនា $\lim\limits_{x\to\infty}\left(\sqrt[3]{x^3+1}-x\right)$

លីមីតនេះមានរាងមិនកំណត់ $\infty-\infty$ កាលណា $x\to\infty$

យើងមាន:

$$\lim_{x\to\infty}\left(\sqrt[3]{x^3+1}-x\right)=\lim_{x\to\infty}\frac{\left(\sqrt[3]{x^3+1}-x\right)\left(\sqrt[3]{(x^3+1)^2}+x\sqrt[3]{x^3+1}+x^2\right)}{\sqrt[3]{(x^3+1)^2}+x\sqrt[3]{x^3+1}+x^2}$$

$$=\lim_{x\to\infty}\frac{(x^3+1)-x^3}{\sqrt[3]{(x^3+1)^2}+x\sqrt[3]{x^3+1}+x^2}=\lim_{x\to\infty}\frac{1}{\sqrt[3]{(x^3+1)^2}+x\sqrt[3]{x^3+1}+x^2}=0$$

ដូច្នេះ: $\lim\limits_{x\to\infty}\left(\sqrt[3]{x^3+1}-x\right)=0$

ឧទាហរណ៍: គណនា $\lim\limits_{x\to\infty}\left(\sqrt[3]{x^3+5x^2}-\sqrt[3]{x^3+8x}\right)$

យើងមាន: $\lim\limits_{x\to\infty}\left(\sqrt[3]{x^3+5x^2}-\sqrt[3]{x^3+8x}\right)$

$$=\lim_{x\to\infty}\frac{\left(\sqrt[3]{x^3+5x^2}-\sqrt[3]{x^3+8x}\right)\left[\sqrt[3]{(x^3+5x)^2}+\sqrt[3]{(x^3+5x^2)(x^3+8x)}+\sqrt[3]{(x^3+8x)^2}\right]}{\sqrt[3]{(x^3+5x)^2}+\sqrt[3]{(x^3+5x^2)(x^3+8x)}+\sqrt[3]{(x^3+8x)^2}}$$

$$=\lim_{x\to\infty}\frac{(x^3+5x^2)-(x^3+8x)}{\sqrt[3]{(x^3+5x)^2}+\sqrt[3]{(x^3+5x^2)(x^3+8x)}+\sqrt[3]{(x^3+8x)^2}}$$

$$=\lim_{x\to\infty}\frac{x^2\left(5-\frac{8}{x}\right)}{x^2\left[\sqrt[3]{\left(1+\frac{5}{x}\right)^2}+\sqrt[3]{\left(1+\frac{5}{x}\right)\left(1+\frac{8}{x^2}\right)}+\sqrt[3]{\left(1+\frac{8}{x^2}\right)^2}\right]}=\frac{5}{3}$$

ដូច្នេះ: $\lim\limits_{x\to\infty}\left(\sqrt[3]{x^3+5x^2}-\sqrt[3]{x^3+8x}\right)=\frac{5}{3}$

---

5. ករណីរាងមិនកំណត់ 1^{∞}

ដើម្បីគណនាលីមីតប្រភេទនេះ គេប្រើ

- រូបមន្ត : $\lim\limits_{\alpha\to 0}(1+\alpha)^{\frac{1}{\alpha}}=\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e$

(e ជាគោលនៃលោការីតនេពែរ ដែល $e\approx 2,718281828$)

- ប្រើវិធី L'hospital ដែលយើងបានរៀបរាប់យ៉ាងពិស្តារក្នុងសៀវភៅ អនុគមន៍អិចស្ប៉ូណង់ស្យែល និងលោការីត ថ្នាក់ទី 10-11 និងវិញ្ញាសារាងមិនកំណត់ផ្សេងទៀត យើងពុំបានលើកយកបង្ហាញក្នុងពេលនេះទេ ព្រោះយើងបានលើកយកបង្ហាញរួចហើយក្នុងសៀវភៅ អិចស្ប៉ូ-លោការីត ម្យ៉ាងទៀតនោះរយៈពេលនេះ យើងខ្ញុំ មាននាទីសាកសួរបំពេញបន្ថែម ខ្លះៗនោះ ដូច្នេះយើងខ្ញុំ សូមលើកយកតែរូបមន្ត ដែលគណនា និង ឧទាហរណ៍ មួយចំនួន

បងប្អូនបានឃើញហើយ៖ ដំណោះស្រាយ លំហាត់ យើងសូម
ឲ្យលទ្ធភាពប្អូនៗ ដែលជាអ្នករៀនរៀន សៀវភៅនេះ
រៀបចំ៖ ម

ឧទាហរណ៍ : គណនា $\lim_{x \to \infty} \left(1 + \frac{K}{x}\right)^x$

កាលណា $x \to \infty$ លីមីតនេះមានរាងមិនកំណត់ 1^∞

យើងបាន:

$$\lim_{x \to \infty} \left(1 + \frac{K}{x}\right)^x = \lim_{x \to \infty} \left[\left(1 + \frac{K}{x}\right)^{\frac{x}{K}}\right]^K = e^K$$

ព្រោះ: $\lim_{x \to \infty} \left(1 + \frac{K}{x}\right)^{\frac{x}{K}} = e$

ដូចនេះ: $\lim_{x \to \infty} \left(1 + \frac{K}{x}\right)^x = e^K$

ឧទាហរណ៍ : គណនា $\lim_{x \to \infty} \left(\frac{x+8}{x-2}\right)^x$

យើងបាន:

$$\lim_{x \to \infty} \left(\frac{x+8}{x-2}\right)^x = \lim_{x \to \infty} \left(\frac{x-2+10}{x-2}\right)^x = \lim_{x \to \infty} \left(1 + \frac{10}{x-2}\right)^x$$

$$= \lim_{x \to \infty} \left[\left(1 + \frac{10}{x-2}\right)^{\frac{x-2}{10}}\right]^{\frac{10x}{x-2}} = e^{\lim_{x \to \infty} \frac{10x}{x-2}} = e^{10}$$

ដូចនេះ: $\lim_{x \to \infty} \left(\frac{x+8}{x-2}\right)^x = e^{10}$

ឧទាហរណ៍: គណនា $\lim_{x \to 0} \frac{\ln(1+mx)}{x}$

យើងបាន:

$$\lim_{x \to 0} \frac{\ln(1+mx)}{x} = \lim_{x \to 0} \frac{1}{x} \ln(1+mx) = \lim_{x \to 0} \ln(1+mx)^{\frac{1}{x}}$$

$$= \ln\left[\lim_{x \to 0} (1+mx)^{\frac{1}{x}}\right] = \ln\left[\lim_{x \to 0} (1+mx)^{\frac{1}{mx} \cdot m}\right]$$

$$= \ln\left[e^m\right] = m \quad (\text{ព្រោះ: } \ln e^n = n)$$

ដូចនេះ: $\lim_{x \to 0} \frac{\ln(1+mx)}{x} = m$

ឧទាហរណ៍: គណនា $\lim_{x \to 0} \frac{\ln(x+2) - \ln 2}{x}$

យើងបាន:

$$\lim_{x \to 0} \frac{\ln(x+2) - \ln 2}{x} = \lim_{x \to 0} \frac{\ln\left(\frac{x}{2} + 1\right)}{x} = \lim_{x \to 0} \frac{1}{x} \ln\left(\frac{x}{2} + 1\right)$$

$$= \lim_{x \to 0} \ln\left(1 + \frac{x}{2}\right)^{\frac{1}{x}}$$

$$= \ln\left[\lim_{x \to 0} \left(1 + \frac{x}{2}\right)^{\frac{1}{x}}\right] = \ln\left[\lim_{x \to 0} \left(1 + \frac{x}{2}\right)^{\frac{2}{x} \cdot \frac{1}{2}}\right]$$

$$= \ln\left[e^{\frac{1}{2}}\right] = \frac{1}{2}$$

ដូចនេះ: $\lim_{x \to 0} \frac{\ln(x+2) - \ln 2}{x} = \frac{1}{2}$

៦. លីមីតនៃអនុគមន៍ត្រីកោណមាត្រ កាលណា $x \to 0$

(x គិតជារ៉ាដ្យង់)

ដើម្បីគណនាលីមីតរាងមិនកំណត់នៃអនុគមន៍ត្រីកោណមាត្រកាលណា $x \to 0$ គេប្រើរូបមន្តខាងក្រោម

$$\lim_{x\to 0} \frac{\sin x}{x} = \lim_{x\to 0} \frac{x}{\sin x} = 1$$

ទូទៅ:

$$\lim_{u\to 0} \frac{\sin u}{u} = \lim_{u\to 0} \frac{u}{\sin u} = 1$$

$$\lim_{x\to 0} \frac{tg x}{x} = \lim_{x\to 0} \frac{x}{tg x} = 1$$

ឧទាហរណ៍: គណនា $\lim\limits_{x\to 0} \dfrac{\sin ax}{\sin bx}$ $(b \neq 0)$

លីមីតនេះមានរាងមិនកំណត់ $\frac{0}{0}$ កាលណា $x \to 0$

ដោះស្រាយ:

$$\lim_{x\to 0} \frac{\sin ax}{\sin bx} = \lim_{x\to 0} \left(\frac{\sin ax}{ax} \cdot \frac{bx}{\sin bx} \cdot \frac{a}{b} \right)$$

$$= \frac{a}{b} \left(\text{ព្រោះ } \lim_{x\to 0} \frac{\sin ax}{ax} = 1 ; \lim_{x\to 0} \frac{bx}{\sin bx} = 1 \right)$$

ដូចនេះ

$$\lim_{x\to 0} \frac{\sin ax}{\sin bx} = \frac{a}{b}$$

ឧទាហរណ៍: គណនា $\lim\limits_{x\to 0} \dfrac{tg x - \sin x}{x^3}$

លីមីតមានរាងមិនកំណត់ $\frac{0}{0}$ កាលណា $x \to 0$

ដោះស្រាយ:

$$\lim_{x\to 0} \frac{tg x - \sin x}{x^3} = \lim_{x\to 0} \frac{tg x (1 - \cos x)}{x^3}$$

$$= \lim_{x\to 0} \frac{tg x \cdot 2\sin^2 \frac{x}{2}}{x^3} \quad \left(\text{ព្រោះ: } 1 - \cos x = 2\sin^2 \tfrac{x}{2} \right)$$

$$= \lim_{x\to 0} \left[\frac{tg x}{x} \left(\frac{\sin \frac{x}{2}}{\frac{x}{2}} \right)^2 \cdot \frac{1}{2} \right] = 1 \cdot 1 \cdot \frac{1}{2} = \frac{1}{2}$$

ដូចនេះ: $\lim\limits_{x\to 0} \dfrac{tg x - \sin x}{x^3} = \dfrac{1}{2}$

ឧទាហរណ៍: គណនា $\lim\limits_{x\to 0} \dfrac{x \sin x}{1 - \cos x}$

ដោះស្រាយ:

$$\lim_{x\to 0} \frac{x \sin x}{1 - \cos x} = \lim_{x\to 0} \frac{2x \sin \frac{x}{2} \cdot \cos \frac{x}{2}}{2\sin^2 \frac{x}{2}}$$

$$= \lim_{x\to 0} \frac{x \cos \frac{x}{2}}{\sin \frac{x}{2}} = \lim_{x\to 0} \left(\frac{\frac{x}{2}}{\sin \frac{x}{2}} \right) 2\cos \frac{x}{2}$$

$$= 1 \cdot 2 = 2$$

ដូចនេះ: $\lim\limits_{x\to 0} \dfrac{x \sin x}{1 - \cos x} = 2$

---

7. លីមីតនៃអនុគមន៍ត្រីកោណមាត្រកាលណា $x \to a$

(x គិតជារ៉ាដ្យង់)

ដើម្បីគណនាលីមីតរាងមិនកំណត់នៃអនុគមន៍ត្រីកោណមាត្រ កាលណា $x \to a$ គេត្រូវ

* តាង $x - a = u$ កាលណា $x \to a$ នោះ $u \to 0$
* ប្រើរូបមន្ត : $\lim\limits_{x \to 0} \dfrac{\sin x}{x} = \lim\limits_{x \to 0} \dfrac{x}{\sin x} = 1$

ឬគេអាចតាង $u = a - x$ ដែល $u \to 0$ កាលណា $x \to a$

ឧទាហរណ៍ : គណនា $\lim\limits_{x \to 1} (1-x) \operatorname{tg} \dfrac{\pi x}{2}$

តាង $u = 1 - x \Leftrightarrow x = 1 - u$

កាលណា $x \to 1$ នោះ $u \to 0$ ដូចនេះ:

$$\lim_{x\to 1}(1-x)\operatorname{tg}\frac{\pi x}{2} = \lim_{u\to 0} u\cdot\operatorname{tg}\left[\frac{\pi}{2}(1-u)\right] = \lim_{u\to 0} u\operatorname{tg}\left[\frac{\pi}{2}-\frac{\pi u}{2}\right]$$

$$= \lim_{u\to 0} u\operatorname{cotg}\frac{\pi u}{2} \quad \left(\text{ព្រោះ: } \operatorname{tg}\left(\frac{\pi}{2}-\alpha\right) = \operatorname{cotg}\alpha\right)$$

$$= \lim_{u\to 0}\frac{u}{\operatorname{tg}\frac{\pi u}{2}} = \lim_{u\to 0}\left(\frac{\frac{\pi u}{2}}{\operatorname{tg}\frac{\pi u}{2}}\cdot\frac{2}{\pi}\right) = 1\cdot\frac{2}{\pi} = \frac{2}{\pi}$$

ដូចនេះ: $\lim\limits_{x\to 1}(1-x)\operatorname{tg}\dfrac{\pi x}{2} = \dfrac{2}{\pi}$

ឧទាហរណ៍: គណនា $\lim\limits_{x\to\pi/4}\dfrac{\cos 2x}{\cos\frac{\pi}{4} - \cos x}$

តាង $u = \dfrac{\pi}{4} - x$ ដែល $u \to 0$ កាលណា $x \to \pi/4$

គេបាន

$$\lim_{x\to\pi/4}\frac{\cos 2x}{\cos\pi/4 - \cos x} = \lim_{u\to 0}\frac{\cos\left(\pi/2 - 2u\right)}{\cos\frac{\pi}{4} - \cos\left(\frac{\pi}{4}-u\right)} = \lim_{u\to 0}\frac{\sin 2u}{-2\sin\frac{u}{2}\cdot\sin\left(\frac{\pi}{4}-\frac{u}{2}\right)}$$

$$= \lim_{u\to 0}\frac{2\sin u\cdot\cos u}{-2\sin\frac{u}{2}\cdot\sin\left(\frac{\pi}{4}-\frac{u}{2}\right)} = \lim_{u\to 0}\frac{4\sin\frac{u}{2}\cos\frac{u}{2}\cos u}{-2\sin\frac{u}{2}\cdot\sin\left(\frac{\pi}{4}-\frac{u}{2}\right)}$$

$$= \lim_{u\to 0}\frac{-2\cos u/2\,\cos u}{\sin\left(\pi/4 - u/2\right)} = \frac{-2\cdot 1\cdot 1}{\frac{\sqrt{2}}{2}} = -2\sqrt{2}$$

ដូចនេះ: $\lim\limits_{x\to\pi/4}\dfrac{\cos 2x}{\cos\pi/4 - \cos x} = -2\sqrt{2}$

ឧទាហរណ៍: គណនា $\lim\limits_{x\to\pi/3}\dfrac{1-2\cos x}{\pi - 3x}$

តាង $u = \pi/3 - x \Leftrightarrow x = \pi/3 - u$ ដែល $u\to 0$ កាលណា $x \to \pi/3$

$$\Rightarrow \lim_{x\to\pi/3}\frac{1-2\cos x}{\pi - 3x} = \lim_{u\to 0}\frac{1-2\cos\left(\pi/3 - u\right)}{\pi - 3\left(\pi/3 - u\right)}$$

$$= \lim_{u\to 0}\frac{1-2\left[\cos\frac{\pi}{3}\cos u + \sin\frac{\pi}{3}\sin u\right]}{3u} = \lim_{u\to 0}\frac{1-2\left[\frac{1}{2}\cos u + \frac{\sqrt{3}}{2}\sin u\right]}{3u}$$

$$= \lim_{u\to 0} \frac{1-\cos u-\sqrt{3}\sin u}{3u} = \lim_{u\to 0} \frac{2\sin^2\frac{u}{2}-2\sqrt{3}\sin\frac{u}{2}\cos\frac{u}{2}}{3u}$$

$$= \lim_{u\to 0} \frac{2\sin\frac{u}{2}\left[\sin\frac{u}{2}-\sqrt{3}\cos\frac{u}{2}\right]}{3u}$$

$$= \lim_{u\to 0} \left(\frac{\sin\frac{u}{2}}{\frac{u}{2}} \cdot \frac{\sin\frac{u}{2}-\sqrt{3}\cos\frac{u}{2}}{3}\right) = 1\cdot\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\sqrt{3}}{3}$$

ដូច្នេះ $\displaystyle\lim_{x\to\frac{\pi}{3}} \frac{1-2\cos x}{\pi-3x} = -\frac{\sqrt{3}}{3}$

---

* **កំណត់សំគាល់**

1. បើ $f(x) \geq g(x)\ ;\ \forall x \in]a-\varepsilon\ ;\ a+\varepsilon[$ នោះ

$$\lim_{x\to a} f(x) \geq \lim_{x\to a} g(x)$$

2. បើ $g(x) \leq f(x) \leq h(x)\ ;\ \forall x \in]a-\varepsilon\ ;\ a+\varepsilon[$

និង $\displaystyle\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = \ell$ នោះគេបាន

$$\lim_{x\to a} f(x) = \ell$$

ឧទាហរណ៍ : គណនា $\displaystyle\lim_{x\to\infty} \frac{\sin x}{x}$

ដោយដឹងថា $\forall x\ :\ -1 \leq \sin x \leq 1$

$$\Rightarrow -\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x} \qquad (x\neq 0)$$

$$\Rightarrow \lim_{x\to\infty}\left(-\frac{1}{x}\right) \leq \lim_{x\to\infty} \frac{\sin x}{x} \leq \lim_{x\to\infty} \frac{1}{x}$$

$$\Leftrightarrow\ 0 \leq \lim_{x\to\infty} \frac{\sin x}{x} \leq 0$$

ដូច្នេះ $\displaystyle\lim_{x\to\infty} \frac{\sin x}{x} = 0$

ភ្នំពេញ ថ្ងៃទី 02.05.1989

.ស. ឃ. ស.

សន្និដ្ឋានទ្រឹស្តីបទទូទៅលើលីមីត

ទ្រឹស្តីបទ ១

បើ $\lim\limits_{x\to a} f(x)=l$; $\lim\limits_{x\to a} g(x)=m$; $\lim\limits_{x\to a} h(x)=n$

នោះគេបាន

$$\boxed{\lim_{x\to a}\left[f(x)+g(x)-h(x)\right]=l+m-n}$$

ឧទាហរណ៍ : គណនាលីមីត : $\lim\limits_{x\to 1}\left(x+2-\frac{1}{x}\right)$

$$\lim_{x\to 1}\left(x+2-\frac{1}{x}\right)=\lim_{x\to 1}x+\lim_{x\to 1}2-\lim_{x\to 1}\frac{1}{x}$$

$$=1+2-1$$

ដូចនេះ $\lim\limits_{x\to 1}\left(x+2-\frac{1}{x}\right)=\underline{2}$

ទ្រឹស្តីបទ ២

បើ $\lim\limits_{x\to a} f(x)=l$; $\lim\limits_{x\to a} g(x)=m$, $\lim\limits_{x\to a} h(x)=n$

នោះគេបាន

$$\boxed{\lim_{x\to a}\left[f(x)\cdot g(x)\cdot h(x)\right]=l\cdot m\cdot n}$$

ឧទាហរណ៍ : គណនាលីមីត $\lim\limits_{x\to 3} 5x^2$

$$\lim_{x\to 3}5x^2=\lim_{x\to 3}5\left(\lim_{x\to 3}x\right)^2=5\cdot 9=\underline{45}$$

ករណីពិសេស

$$\boxed{\begin{array}{l}
* \ \lim\limits_{x\to a}\left[f(x)\right]^n=\left[\lim\limits_{x\to a}f(x)\right]^n=l^n\ ;\ n\in\mathbb{Q} \\
* \ \lim\limits_{x\to a}\lambda f(x)=\lambda\lim\limits_{x\to a}f(x)\ ;\ \lambda \text{ ថេរ} \\
* \ \lim\limits_{x\to a}\sqrt{f(x)}=\sqrt{\lim\limits_{x\to a}f(x)}\ ;\ f(x)\geq 0\Rightarrow\lim f(x)\geq 0
\end{array}}$$

ឧទាហរណ៍ : គណនាលីមីត

1. $\lim\limits_{x\to 2}\sqrt{2x+3x^2}$

$$\lim_{x\to 2}\sqrt{2x+3x^2}=\sqrt{\lim_{x\to 2}2x+3x^2}=\sqrt{\lim_{x\to 2}2x+\lim_{x\to 2}3x^2}$$

$$=\sqrt{4+12}=4$$

2. $\lim\limits_{x\to 2}4x^3$

$$\lim_{x\to 2}4x^3=4\lim_{x\to 2}x^3=4\cdot 8=32$$

3. $\lim\limits_{x\to 1}\left(5x^2+x-\sqrt{4x}\right)$

$$\lim_{x\to 1}\left(5x^2+x-\sqrt{4x}\right)=\lim_{x\to 1}5x^2+\lim_{x\to 1}x-\lim_{x\to 1}\sqrt{4x}$$

$$=5+1-2=4$$

ទ្រឹស្តីបទ៣

បើ $\lim\limits_{x\to a}(f(x)) = l$

ក. $\lim\limits_{x\to a}\dfrac{1}{f(x)} = \dfrac{1}{l}$, កាលណា $l \neq 0$

ខ. $\lim\limits_{x\to a}\dfrac{1}{f(x)} = 0$, កាលណា $l \to \pm\infty$

គ $\lim\limits_{x\to a}\dfrac{1}{f(x)} = +\infty$, កាលណា $l \to 0^+$

ឧទាហរណ៍ : គណនាលីមីត $\lim\limits_{x\to 0}\dfrac{2x+4}{x}$

$$\lim_{x\to 0}\frac{2x+4}{x} = \lim_{x\to 0} 2 + \lim_{x\to 0}\frac{4}{x} = 2+\infty = \infty$$

ទ្រឹស្តីបទ4

បើ $\lim\limits_{x\to a} f(x) = l$; $\lim\limits_{x\to a} g(x) = m.$

យើងបាន

$$\boxed{\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{\lim\limits_{x\to a} f(x)}{\lim\limits_{x\to a} g(x)} = \frac{l}{m}} \quad m \neq 0$$

ករណីពិសេស

- $\lim\limits_{x\to a}\dfrac{f(x)}{g(x)} = 0$, កាលណា $l \neq 0$ ហើយ $m \to \infty$

$$\lim_{x\to a}\left|\frac{f(x)}{g(x)}\right| = \infty,$$ កាលណា $l \neq 0$ ហើយ $m = 0$

ឧទាហរណ៍ : គណនាលីមីត

1. $\lim\limits_{x\to 1}\dfrac{2x+3}{x-5}$

$$\lim_{x\to 1}\frac{2x+3}{x-5} = \frac{\lim\limits_{x\to 1} 2x+3}{\lim\limits_{x\to 1} x-5} = -\frac{5}{4}$$

2. $\lim\limits_{x\to 2}\dfrac{x+1}{x-2}$

$$\lim_{x\to 2}\frac{x+1}{x-2} = \frac{\lim\limits_{x\to 2} x+1}{\lim\limits_{x\to 2} x-2} = \frac{3}{0} = \infty$$

លំហាត់

លំហាត់ដែលមានរាងមិនកំណត់

I ចូររកលីមីតខាងក្រោម

1. $\lim\limits_{x\to -\frac{1}{3}} \dfrac{9x^2-1}{3x+1}$; 2 $\lim\limits_{x\to 1} \dfrac{x^2+x-2}{x-1}$

3 $\lim\limits_{x\to -2} \dfrac{x+2}{x^2+4x+4}$; 4 $\lim\limits_{x\to 1} \dfrac{\sqrt{x+3}-2}{x-1}$

5 $\lim\limits_{x\to 2} \dfrac{\sqrt{x^2-4}}{x^3-x^2-x-2}$; 6 $\lim\limits_{x\to 4} \dfrac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}$

7 $\lim\limits_{x\to 0} \dfrac{\sqrt{1+x}-\sqrt[3]{1+x}}{x}$; 8 $\lim\limits_{x\to 0} \dfrac{\sqrt{x+1}+\sqrt{x+4}-3}{x}$

9 $\lim\limits_{x\to 1} \dfrac{\sqrt[3]{x}-1}{\sqrt[3]{4x+4}-2}$; 10 $\lim\limits_{h\to 0} \dfrac{(x+h)^3-x^3}{h}$

សំរាយបញ្ជាក់

1 $\lim\limits_{x\to -\frac{1}{3}} \dfrac{9x^2-1}{3x+1}$ មានរាងមិនកំណត់ $\frac{0}{0}$

$$\frac{9x^2-1}{3x+1} = \frac{(3x+1)(3x-1)}{3x+1} = 3x-1 \quad (x \neq -\tfrac{1}{3})$$

$$\lim_{x\to -\frac{1}{3}} \frac{9x^2-1}{3x+1} = \lim_{x\to -\frac{1}{3}} (3x-1) = -2$$

$$\boxed{\lim_{x\to -\frac{1}{3}} \frac{9x^2-1}{3x+1} = -2}$$

2 $\lim\limits_{x\to 1} \dfrac{x^2+x-2}{x-1}$ មានរាងមិនកំណត់ $\frac{0}{0}$

$$\frac{x^2+x-2}{x-1} = \frac{(x-1)(x+2)}{x-1} = x+2 \quad (x \neq 1)$$

$$\lim_{x\to 2} \frac{x^2+x-2}{x-1} = \lim_{x\to 1} (x+2) = 3$$

$$\boxed{\lim_{x\to 1} \frac{x^2+x-2}{x-1} = 3}$$

3 $\lim\limits_{x\to -2} \dfrac{x+2}{x^2+4x+4}$, មានរាងមិនកំណត់ $\frac{0}{0}$

$$\frac{x+2}{x^2+4x+4} = \frac{x+2}{(x+2)^2} = \frac{1}{x+2} \quad (x \neq -2)$$

$$\lim_{x\to -2} \frac{x+2}{x^2+4x+4} = \lim_{x\to -2} \frac{1}{x+2} = \infty$$

$$\boxed{\lim_{x\to -2} \frac{x+2}{x^2+4x+4} = \infty}$$

5. $\lim\limits_{x\to 2} \dfrac{\sqrt{x^2-4}}{x^3-x^2-x-2}$, មានរាងមិនកំណត់ $\frac{0}{0}$

$$\frac{\sqrt{x^2-4}}{x^3-x^2-x-2} \quad \left(\text{មានន័យកាលណា } \mathcal{D}=]-\infty,-2[\,\cup\,]2,+\infty[\right)$$

$$=\frac{x^2-4}{(x-2)(x^2+x+1)\sqrt{x^2-4}}=\frac{x+2}{(x^2+x+1)\sqrt{x^2-4}} \quad (x\neq 2)$$

$$\lim_{x\to 2}\frac{\sqrt{x^2-4}}{x^3-x^2-x-2}=\lim_{x\to 2}\frac{x+2}{(x^2+x+1)\sqrt{x^2-4}}=\frac{4}{0}=\infty$$

$$\boxed{\lim_{x\to 2}\frac{x^2-4}{x^3-x^2-x-2}=\infty}$$

4. $\lim\limits_{x\to 1}\dfrac{\sqrt{x+3}-2}{x-1}$, មានរាងមិនកំណត់ $\frac{0}{0}$

$$\frac{\sqrt{x+3}-2}{x-1} \quad \left(\text{មានន័យកាលណា } \mathcal{D}=[-3,+\infty[-\{1\}\right)$$

$$\frac{\sqrt{x+3}-2}{x-1}=\frac{x+3-4}{(x-1)(\sqrt{x+3}+2)}=\frac{x-1}{(x-1)(\sqrt{x+3}+2)}=\frac{1}{\sqrt{x+3}+2}$$
$(x\neq 1)$

$$\lim_{x\to 1}\frac{\sqrt{x+3}-2}{x-1}=\lim_{x\to 1}\frac{1}{\sqrt{x+3}+2}=\frac{1}{4}$$

$$\boxed{\lim_{x\to 1}\frac{\sqrt{x+3}-2}{x-1}=\frac{1}{4}}$$

6 $\lim\limits_{x\to 3}\dfrac{(x-1)-\sqrt{x+1}}{x^2-3x}$, មានរាងមិនកំណត់ $\frac{0}{0}$

$$\frac{(x-1)-\sqrt{x+1}}{x^2-3x} \quad \left(\text{មានន័យលុះត្រាតែ } \mathcal{D}=[-1,+\infty[-\{0,3\}\right)$$

$$=\frac{(x-1)^2-(x+1)}{(x^2-3x)(x-1+\sqrt{x+1})}=\frac{x^2-3x}{(x^2-3x)(x-1+\sqrt{x+1})}, \quad x\neq 0;\ x\neq 3$$

$$=\frac{1}{x-1+\sqrt{x+1}}$$

$$\lim_{x\to 3}\frac{(x-1)-\sqrt{x+1}}{x^2-3x}=\lim_{x\to 3}\frac{1}{x-1+\sqrt{x+1}}=\frac{1}{4}$$

$$\boxed{\lim_{x\to 3}\frac{(x-1)-\sqrt{x+1}}{x^2-3x}=\frac{1}{4}}$$

7. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt[3]{1+x}}{x}$; មានរាងមិនកំណត់ $\frac{0}{0}$

$$\frac{\sqrt{1+x}-\sqrt[3]{1+x}}{x}, \quad \text{មានន័យលុះត្រាតែ } \mathcal{D}=]-1,+\infty[-\{0\}$$

$$=\frac{(\sqrt{1+x})^6-(\sqrt[3]{1+x})^6}{x\left[(\sqrt{1+x})^5+(\sqrt{1+x})^4\cdot\sqrt[3]{1+x}+\dots+(\sqrt[3]{1+x})^5\right]}=\frac{N}{D}$$

$$N=(1+x)^3-(1+x)^2=(1+x)^2(1+x-1)=x(1+x)^2$$

$$\frac{N}{D}=\frac{(1+x)^2}{(\sqrt{1+x})^5+(\sqrt{1+x})^4\cdot\sqrt{1+x}+\dots+(\sqrt[3]{1+x})^5}$$

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt[3]{1+x}}{x}=\lim_{x\to 0}\frac{(1+x)^2}{(\sqrt{1+x})^5+(\sqrt{1+x})^4\sqrt{1+x}+\dots+(\sqrt[3]{1+x})^5}$$

$$\boxed{\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt[3]{1+x}}{x}=\frac{1}{6}}$$

8. $\lim\limits_{x\to 0}\dfrac{\sqrt{x+1}+\sqrt{x+4}-3}{x}$, មានរាងមិនកំណត់ $\frac{0}{0}$

$\dfrac{\sqrt{x+1}+\sqrt{x+4}-3}{x}$ មាននិយមន័យលុះត្រាតែ $\mathcal{D}=]-1,+\infty[-\{0\}$

$$=\frac{\left(\sqrt{(x+1)}+\sqrt{x+4}\right)^2-9}{x\left[\left(\sqrt{x+1}+\sqrt{x+4}\right)+3\right]}=\frac{x+1+x+4+2\sqrt{(x+1)(x+4)}-9}{x\left[\left(\sqrt{x+1}+\sqrt{x+4}\right)+3\right]}$$

$$=\frac{2x\ \ 4+2\sqrt{(x+1)(x+4)}}{x\left[\left(\sqrt{x+1}+\sqrt{x+4}\right)+3\right]}=\frac{2\left(x-2+\sqrt{(x+1)(x+4)}\right)}{x\left[\left(\sqrt{x+2}+\sqrt{x+4}\right)+3\right]}$$

$$=\frac{2\left[(x-2)^2-(x+1)(x+4)\right]}{x\left[\left(\sqrt{x+1}+\sqrt{x+4}+3\right]\left[(x-2)-\sqrt{(x+1)(x+4)}\right]}$$

$$=\frac{2\left(x^2+4-4x-x^2-4x-x-4\right)}{x\left[\left(\sqrt{x+1}+\sqrt{x+4}+3\right]\left[(x-2)-\sqrt{(x+1)(x+4)}\right]}$$

$$=\frac{-18}{\left[\left(\sqrt{x+1}+\sqrt{x+4}+3\right]\left[(x-2)-\sqrt{(x+1)(x+4)}\right]}\quad x\neq 0$$

$$\lim_{x\to 0}\frac{\sqrt{x+1}+\sqrt{x+4}-3}{x}=\lim_{x\to 0}\frac{-18}{\left[\left(\sqrt{x+1}+\sqrt{x+4}+3\right]\left[(x-2)-\sqrt{(x+1)(x+4)}\right]}$$

$$=\frac{18}{24}=\frac{3}{4}$$

$$\boxed{\lim_{x\to 0}\frac{\sqrt{x+1}+\sqrt{x+4}-3}{x}=\frac{3}{4}}$$

9. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x}-1}{\sqrt[3]{4x+4}-2}$ មានរាងមិនកំណត់ $\frac{0}{0}$

$\dfrac{\sqrt[3]{x}-1}{\sqrt[3]{4x+4}-2}$ (មាននិយមន័យលុះត្រាតែ $\mathcal{D}=R-\{-1\}$)

$$=\frac{(x-1)\left[\left(\sqrt[3]{4x+4}\right)^2+2\sqrt[3]{4x+4}+4\right]}{(4x+4-8)\left[\left(\sqrt[3]{x}\right)^2+\sqrt[3]{x}+1\right]}=\frac{(x-1)\left[\left(\sqrt[3]{4x+4}\right)^2+2\sqrt[3]{4x+4}+4\right]}{4(x-1)\left[\left(\sqrt[3]{x}\right)^2+\sqrt[3]{x}+1\right]}$$

$$=\frac{\left(\sqrt[3]{4x+4}\right)^2+2\sqrt[3]{4x+4}+4}{4\left[\left(\sqrt[3]{x}\right)^2+\sqrt[3]{x}+1\right]}\ ,\quad x\neq 1$$

$$\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt[3]{4x+4}-2}=\lim_{x\to 1}\frac{\left(\sqrt[3]{4x+4}\right)^2+2\sqrt[3]{4x+4}+4}{4\left[\left(\sqrt[3]{x}\right)^2+\sqrt[3]{x}+1\right]}=\frac{4\times 4\times 4}{4\times 4\times 4}$$

$$=1$$

$$\boxed{\lim_{x\to 1}\frac{\sqrt[3]{x}-1}{\sqrt[3]{4x+4}-2}=1}$$

10. $\lim\limits_{h\to 0}\dfrac{(x+h)^3-x^3}{h}$ មានរាងមិនកំណត់ $\frac{0}{0}$

$$\frac{(x+h)^3-x^3}{h}=\frac{(x+h-x)\left[(x+h)^2+x(x+h)+x^2\right]}{h}$$

$$=\frac{h\left(x^2+2xh+h^2+xh+x^2+x^2\right)}{h}$$

$$=3x^2+3xh+h^2$$

$$\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}=\lim_{h\to 0}3x^2+3xh+h^2=3x^2$$

$$\boxed{\lim_{h\to 0}\frac{(x+h)^3-x^3}{h}=3x^2}$$

II ស្វែងរកលីមីតខាងក្រោម

11 $\lim\limits_{x\to\infty} \dfrac{8x^3+12x^2+x+1}{6x^3+3x^2-5x+2}$; 12 $\lim\limits_{x\to\infty} \dfrac{2x^2-3x+1}{x^3+2x+5}$

13 $\lim\limits_{x\to\infty} \dfrac{3x^2+2x+5}{2x+1}$; 14 $\lim\limits_{|x|\to\infty} \dfrac{\sqrt{x^2-1}}{x}$

15 $\lim\limits_{x\to\infty} \dfrac{x^3-4x^2+5x-1}{2x^3+3x^2-4x+6}$; 16 $\lim\limits_{x\to\infty} \dfrac{x^2+3x-5}{2x^2+1}$

17 $\lim\limits_{x\to\infty} \dfrac{x^3+5x-7}{x^2+3x-1}$, 18 $\lim\limits_{x\to\infty} \dfrac{x+5}{2x^2+3x+7}$

19 $\lim\limits_{x\to\infty} \dfrac{x^4-5x}{x^2-3x+1}$, 20 $\lim\limits_{x\to\infty} \dfrac{1+x-3x^3}{1+x^2+3x^3}$

សំរាយបញ្ជាក់

11 $\lim\limits_{x\to\infty} \dfrac{8x^3+12x^2+x+1}{6x^3+3x^2-5x+2}$, មានរាងមិនកំណត់ $\dfrac{\infty}{\infty}$

$$\frac{8x^3+12x^2+x+1}{6x^3+3x^2-5x+2} = \frac{x^3\left(8+\frac{12}{x}+\frac{1}{x^2}+\frac{1}{x^3}\right)}{x^3\left(6+\frac{3}{x}-\frac{5}{x^2}+\frac{2}{x^3}\right)}$$

$$= \frac{8+\frac{12}{x}+\frac{1}{x^2}+\frac{1}{x^3}}{6+\frac{3}{x}-\frac{5}{x^2}+\frac{2}{x^3}} \; ; \; x \neq 0$$

$$\lim_{x\to\infty} \frac{8x^3+12x^2+x+1}{6x^3+3x^2-5x+2} = \lim_{x\to\infty} \frac{8+\frac{12}{x}+\frac{1}{x^2}+\frac{1}{x^3}}{6+\frac{3}{x}-\frac{5}{x^2}+\frac{2}{x^3}} = \frac{8}{6} = \frac{4}{3}$$

$$\boxed{\lim_{x\to\infty} \frac{8x^3+12x^2+x+1}{6x^3+3x^2-5x+2} = \frac{4}{3}}$$

12. $\lim\limits_{x\to\infty} \dfrac{2x^2-3x+1}{x^3+2x+5}$, មានរាងមិនកំណត់ $\dfrac{\infty}{\infty}$

$$\frac{2x^2-3x+1}{x^3+2x+5} = \frac{x^2\left(1-\frac{3}{x}+\frac{1}{x^2}\right)}{x^3\left(1+\frac{2}{x^2}+\frac{5}{x^3}\right)} = \frac{1-\frac{3}{x}+\frac{1}{x^2}}{x\left(1+\frac{2}{x^2}+\frac{5}{x^3}\right)} \; ; \; x \neq 0$$

$$\lim_{x\to\infty} \frac{2x^2-3x+1}{x^3+2x+5} = \lim_{x\to\infty} \frac{1-\frac{3}{x}+\frac{1}{x^2}}{x\left(1+\frac{2}{x^2}+\frac{5}{x^3}\right)} = 0$$

$$\boxed{\lim_{x\to\infty} \frac{2x^2-3x+1}{x^3+2x+5} = 0}$$

13 $\lim\limits_{x\to\infty} \dfrac{3x^2+2x+5}{2x+1}$, មានរាងមិនកំណត់ $\dfrac{\infty}{\infty}$

$$\frac{3x^2+2x+5}{2x+1} = \frac{x^2\left(3+\frac{2}{x}+\frac{5}{x^2}\right)}{x\left(2+\frac{1}{x}\right)} = \frac{x\left(3+\frac{2}{x}+\frac{5}{x^2}\right)}{2+\frac{1}{x}} \; , \; x \neq 0$$

$$\lim_{x\to\infty} \frac{3x^2+2x+5}{2x+1} = \lim_{x\to\infty} \frac{x\left(3+\frac{2}{x}+\frac{5}{x^2}\right)}{2+\frac{1}{x}} = \infty$$

$$\boxed{\lim_{x\to\infty} \frac{3x^2+2x+5}{2x+1} = \infty}$$

14 $\lim\limits_{|x|\to\infty} \dfrac{\sqrt{x^2-1}}{x}$; មានរាងមិនកំណត់ $\dfrac{\infty}{\infty}$

$\dfrac{\sqrt{x^2-1}}{x}$ មាននិយមន័យលុះត្រាតែ $D =]-\infty,-1] \cup [1,+\infty[$

$$\frac{\sqrt{x^2-1}}{x} = \frac{\sqrt{x^2(1-\frac{1}{x^2})}}{x} = \frac{|x|}{x}\sqrt{1-\frac{1}{x^2}}$$

+ បើ $x>0 \Rightarrow |x| = x$, $\lim\limits_{x\to+\infty}\sqrt{1-\dfrac{1}{x^2}} = 1$

+ បើ $x<0 \Rightarrow |x| = -x$, $\lim\limits_{x\to-\infty}\left(-\sqrt{1-\dfrac{1}{x^2}}\right) = -1$

15 $\lim\limits_{x\to\infty} \dfrac{x^3-4x^2+5x-1}{2x^3+3x^2-4x+6}$ មានរាងមិនកំណត់ $\dfrac{\infty}{\infty}$

$$\frac{x^3-4x^2+5x-1}{2x^3+3x^2-4x+6} = \frac{x^3\left(1-\frac{4}{x}+\frac{5}{x^2}-\frac{1}{x^3}\right)}{x^3\left(2+\frac{3}{x}-\frac{4}{x^2}+\frac{6}{x^3}\right)} = \frac{1-\frac{4}{x}+\frac{5}{x^2}-\frac{1}{x^3}}{2+\frac{3}{x}-\frac{4}{x^2}+\frac{6}{x^3}}$$

$$\lim_{x\to\infty}\frac{x^3-4x^2+5x-1}{2x^3+3x^2-4x+6} = \lim_{x\to\infty}\frac{1-\frac{4}{x}+\frac{5}{x^2}-\frac{1}{x^3}}{2+\frac{3}{x}-\frac{4}{x^2}+\frac{6}{x^3}} = \frac{1}{2}$$

$$\boxed{\lim_{x\to\infty}\frac{x^3-4x^2+5x-1}{2x^3+3x^2-4x+6} = \frac{1}{2}}$$

16 $\lim\limits_{x\to\infty} \dfrac{x^2+3x-5}{2x^2+1}$, មានរាងមិនកំណត់ $\dfrac{\infty}{\infty}$

$$\frac{x^2+3x-5}{2x^2+1} = \frac{x^2\left(1+\frac{3}{x}-\frac{5}{x^2}\right)}{x^2\left(2+\frac{1}{x^2}\right)} = \frac{1+\frac{3}{x}-\frac{5}{x^2}}{2+\frac{1}{x^2}} \quad (x\neq 0)$$

$$\lim_{x\to\infty}\frac{x^2+3x-5}{2x^2+1} = \lim_{x\to\infty}\frac{1+\frac{3}{x}-\frac{5}{x^2}}{2+\frac{1}{x^2}} = \frac{1}{2}$$

$$\boxed{\lim_{x\to\infty}\frac{x^2+3x-5}{2x^2+1} = \frac{1}{2}}$$

17 $\lim\limits_{x\to\infty} \dfrac{x^3+5x-7}{x^2+3x-1}$, មានរាងមិនកំណត់ $\dfrac{\infty}{\infty}$

$$\frac{x^3+5x-7}{x^2+3x-1} = \frac{x^3\left(1+\frac{5}{x^2}-\frac{7}{x^3}\right)}{x^2\left(1+\frac{3}{x}-\frac{1}{x^2}\right)} = \frac{x\left(1+\frac{5}{x^2}-\frac{7}{x^3}\right)}{1+\frac{3}{x}-\frac{1}{x^2}} \quad (x\neq 0)$$

$$\lim_{x\to\infty}\frac{x^3+5x-7}{x^2+3x-1} = \lim_{x\to\infty}\frac{x\left(1+\frac{5}{x^2}-\frac{7}{x^3}\right)}{1+\frac{3}{x}-\frac{1}{x^2}} = \infty$$

$$\boxed{\lim_{x\to\infty}\frac{x^3+5x-7}{x^2+3x-1} = \infty}$$

18 $\lim\limits_{x\to\infty} \dfrac{x+5}{2x^2+3x+7}$, មានរាងមិនកំណត់ $\dfrac{\infty}{\infty}$

$$\frac{x+5}{2x^2+3x+7} = \frac{x\left(1+\frac{5}{x}\right)}{x^2\left(2+\frac{3}{x}+\frac{7}{x^2}\right)} = \frac{1+\frac{5}{x}}{x\left(2+\frac{3}{x}+\frac{7}{x^2}\right)} \quad (x\neq 0)$$

$$\lim_{x\to\infty}\frac{x+5}{2x^2+3x+7} = \lim_{x\to\infty}\frac{1+\frac{5}{x}}{x\left(2+\frac{3}{x}+\frac{7}{x^2}\right)} = 0$$

$$\boxed{\lim_{x\to\infty}\frac{x+5}{2x^2+3x+7} = 0}$$

19 $\lim\limits_{x\to\infty} \dfrac{x^4-5x}{x^2-3x+1}$, មានរាងមិនកំណត់ $\dfrac{\infty}{\infty}$

$$\frac{x^4-5x}{x^2-3x+1} = \frac{x^4\left(1-\frac{5}{x^3}\right)}{x^2\left(1-\frac{3}{x}+\frac{1}{x^2}\right)} = \frac{x^2\left(1-\frac{5}{x^3}\right)}{1-\frac{3}{x}+\frac{1}{x^2}} \quad (x\neq 0)$$

$$\lim_{x\to\infty} \frac{x^4-5x}{x^2-3x+1} = \lim_{x\to\infty} \frac{x^2\left(1-\frac{5}{x^3}\right)}{1-\frac{3}{x}+\frac{1}{x^2}} = +\infty$$

$$\boxed{\lim_{x\to\infty} \frac{x^4-5x}{x^2-3x+1} = +\infty}$$

20. $\lim\limits_{x\to\infty} \dfrac{1+x-3x^3}{1+x^2+3x^3}$ មានរាងមិនកំណត់ $\dfrac{\infty}{\infty}$

$$\frac{1+x-3x^3}{1+x^2+3x^3} = \frac{x^3\left(\frac{1}{x^3}+\frac{1}{x^2}-3\right)}{x^3\left(\frac{1}{x^3}+\frac{1}{x}+3\right)} = \frac{\frac{1}{x^3}+\frac{1}{x^2}-3}{\frac{1}{x^3}+\frac{1}{x}+3} \quad (x\neq 0)$$

$$\lim_{x\to\infty} \frac{1+x-3x^3}{1+x^2+3x^3} = \lim_{x\to\infty} \frac{\frac{1}{x^3}+\frac{1}{x^2}-3}{\frac{1}{x^3}+\frac{1}{x}+3} = -1$$

$$\boxed{\lim_{x\to\infty} \frac{1+x-3x^3}{1+x^2+3x^3} = -1}$$

II ព្យាករណ៍លីមីតការណ៍រួម

21. $\lim\limits_{x\to\pm\infty}\left(\sqrt{x^2-1}-x\right)$; 22 $\lim\limits_{x\to\pm\infty}\left(\sqrt{x^2+3x}-\sqrt{x^2+1}\right)$

23. $\lim\limits_{n\to\infty}\left[\sqrt[3]{m^3-2n^2}-n\right]$; 24. $\lim\limits_{n\to\infty} n\left(\sqrt{n^2+1}-\sqrt{n^2-2}\right)$

25 $\lim\limits_{x\to\infty}\left(\sqrt{ax^2+bx+c}-x\sqrt{a}\right) \quad (a>0)$

26 $\lim\limits_{x\to+\infty}\left(\sqrt[3]{x^3+3x^2}-\sqrt{x^2-2x}\right)$

27 $f(x)=\left(\sqrt{x^2+4x+7}-\sqrt{x^2-4}\right)$ រក $\lim\limits_{x\to\pm\infty} f(x)$

28. $\lim\limits_{x\to\infty}\left(\sqrt[3]{x^3+x+1}-\sqrt[3]{x^3-x-1}\right)$; 29 $\lim\limits_{x\to\infty}\left(\sqrt{x+1}-\sqrt{x}\right)$

30 $\lim\limits_{x\to\infty}\left(\sqrt[3]{x^3+x+1}-\sqrt{x^2-x-1}\right)$

សំរាយបញ្ជាក់

លំហាត់21 $\lim\limits_{x\to\pm\infty}\left(\sqrt{x^2-1}-x\right)$: មានចំលើយ

* $\lim\limits_{x\to-\infty}\left(\sqrt{x^2-1}-x\right) = +\infty+\infty = +\infty$
* $\lim\limits_{x\to+\infty}\left(\sqrt{x^2-1}-x\right)$: លីមីតរាងមិនកំណត់ $\infty-\infty$

ដោយប្រើកន្សោមឆ្លាស់លើនិងក្រោម

$$\lim_{x\to+\infty}\left(\sqrt{x^2-1}-x\right) = \lim_{x\to+\infty} \frac{-1}{\sqrt{x^2-1}+x} = 0$$

ដូចនេះ

$$\boxed{\lim_{x\to\pm\infty}\left(\sqrt{x^2-1}-x\right)=\begin{cases}+\infty & : \text{បើ } x\to-\infty\\ 0 & : \text{បើ } x\to+\infty\end{cases}}$$

22 $\lim\limits_{x\to\pm\infty}\left(\sqrt{x^2+3x}-\sqrt{x^2+1}\right)$, មានរាងមិនកំណត់ $\infty-\infty$

$\sqrt{x^2+3x}-\sqrt{x^2+1}$ មានន័យលុះត្រាតែ $D=\left]-\infty,-3\right]\cup\left[0,+\infty\right[$

$$\sqrt{x^2+3x}-\sqrt{x^2+1}=\frac{\left(\sqrt{x^2+3x}-\sqrt{x^2+1}\right)\left(\sqrt{x^2+3x}+\sqrt{x^2+1}\right)}{\left(\sqrt{x^2+3x}+\sqrt{x^2+1}\right)}$$

$$=\frac{x^2+3x-x^2-1}{\sqrt{x^2+3x}+\sqrt{x^2+1}}=\frac{x-\left(3-\frac{1}{x}\right)}{x^2\left(1+\frac{3}{x}\right)+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}$$

$$=\frac{x\left(3-\frac{1}{x}\right)}{|x|\left(\sqrt{1+\frac{3}{x}}+\sqrt{1+\frac{1}{x^2}}\right)}$$

* បើ $x>0$ នោះ $|x|=x$

$$\lim_{x\to+\infty}\left(\sqrt{x^2+3x}-\sqrt{x^2+1}\right)=\lim_{x\to+\infty}\frac{x\left(3-\frac{1}{x}\right)}{x\left(\sqrt{1+\frac{3}{x}}+\sqrt{1+\frac{1}{x^2}}\right)}$$

$$=\lim_{x\to+\infty}\frac{3x}{2x}=\frac{3}{2}$$

$$\boxed{\lim_{x\to+\infty}\left(\sqrt{x^2+3x}-\sqrt{x^2+1}\right)=\frac{3}{2}}$$

* បើ $x<0$ នោះ $|x|=-x$

$$\lim_{x\to-\infty}\left(\sqrt{x^2+3x}-\sqrt{x^2+1}\right)=\lim_{x\to-\infty}\frac{x\left(3-\frac{1}{x}\right)}{-x\left(\sqrt{1+\frac{3}{x}}+\sqrt{1+\frac{1}{x^2}}\right)}$$

$$=\lim_{x\to-\infty}\frac{3x}{-2x}=-\frac{3}{2}$$

$$\boxed{\lim_{x\to-\infty}\left(\sqrt{x^2+3x}-\sqrt{x^2+1}\right)=-\frac{3}{2}}$$

23 $\lim\limits_{n\to\infty}\left[\sqrt[3]{n^3-2n^2}-n\right]$ មានរាងមិនកំណត់ $\infty-\infty$

$$\sqrt[3]{n^3-2n^2}-n=\frac{n^3-2n^2-n^3}{\left(\sqrt[3]{n^3-2n^2}\right)^2+n\sqrt[3]{n^3-2n^2}+n^2}$$

$$=\frac{-2n^2}{n^2\left[\sqrt[3]{\left(1-\frac{2}{n}\right)^2}+\sqrt[3]{1-\frac{2}{n}}+1\right]}$$

$$\frac{-2}{\sqrt[3]{\left(1-\frac{2}{n}\right)}+\sqrt[3]{1-\frac{2}{n}+1}}$$

$$\lim_{n\to\infty}\left[\sqrt[3]{n^3-2n^2}-n\right]=\lim_{n\to\infty}\frac{-2}{\sqrt[3]{\left(1-\frac{2}{n}\right)^2}+\sqrt[3]{1-\frac{2}{n}}+1}=-\frac{2}{3}$$

$$\boxed{\lim_{n\to\infty}\left(\sqrt[3]{n^3-2n^2}-n\right)=-\frac{2}{3}}$$

24. $\lim\limits_{n\to\infty} n\left[\sqrt{n^2+1}-\sqrt{n^2-2}\right]$, មានរាងមិនកំណត់ $\infty-\infty$

$$n\left[\sqrt{n^2+1}-\sqrt{n^2-2}\right]=\frac{n[(n^2+1)-(n^2-2)]}{\sqrt{n^2+1}+\sqrt{n^2-2}}=\frac{n(n^2+1-n^2+2)}{\sqrt{n^2+1}+\sqrt{n^2-2}}$$

$$=\frac{3n}{n\left(\sqrt{1+\frac{1}{n^2}}+\sqrt{1-\frac{2}{n^2}}\right)}=\frac{3}{\sqrt{1+\frac{1}{n^2}}+\sqrt{1-\frac{2}{n^2}}}\quad (n\neq 0)$$

$$\lim_{n\to\infty} n\left[\sqrt{n^2+1}-\sqrt{n^2-2}\right]=\lim_{n\to\infty}\frac{3}{\sqrt{1+\frac{1}{n^2}}+\sqrt{1-\frac{2}{n^2}}}=\frac{3}{2}$$

$$\boxed{\lim_{n\to\infty} n\left[\sqrt{n^2+1}-\sqrt{n^2-2}\right]=\frac{3}{2}}$$

25. $\lim\limits_{x\to\infty}\left(\sqrt{ax^2+bx+c}-x\sqrt{a}\right)$ $(x>0)$ មានរាងមិនកំណត់ $\infty-\infty$

$$\sqrt{ax^2+bx+c}-x\sqrt{a}=\frac{ax^2+bx+c-ax^2}{\sqrt{ax^2+bx+c}+x\sqrt{a}}=\frac{bx+c}{x\left(\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\sqrt{a}\right)}$$

$$=\frac{b+\frac{c}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\sqrt{a}}\qquad x\neq 0$$

$$\lim_{x\to\infty}\left(\sqrt{ax^2+bx+c}-x\sqrt{a}\right)=\lim_{x\to\infty}\frac{b+\frac{c}{x}}{\sqrt{a+\frac{b}{x}+\frac{c}{x^2}}+\sqrt{a}}=\frac{b}{2\sqrt{a}}$$

$$\boxed{\lim_{x\to\infty}\left(\sqrt{ax^2+bx+c}-x\sqrt{a}\right)=\frac{b}{2\sqrt{a}}}$$

លំហាត់ទី 26

$\lim\limits_{x\to+\infty}\left(\sqrt[3]{x^3+3x^2}-\sqrt{x^2-2x}\right)$ មានរាងមិនកំណត់ $\infty-\infty$

រំលឹក $a^6-b^6=(a-b)(a^5-a^4b+a^3b^2+a^2b^3+ab^4+b^5)$

$$\sqrt[3]{x^3+3x^2}-\sqrt{x^2-2x}=\frac{(x^3+3x^2)^2-(x^2-2x)^3}{\sqrt[3]{(x^3+3x^2)^5}+\sqrt[3]{(x^3+3x^2)^4}\cdot\sqrt{x^2-2x}+\dots+\sqrt{(x^2-2x)^5}}$$

$$=\frac{N}{D}$$

$$N=(x^3+3x^2)^2-(x^2-2x)^3$$

$$=x^6+9x^4+6x^5-(x^6-6x^5+12x^4-8x^3)$$

$$=12x^5-3x^4+8x^3=x^5\left(12-\frac{3}{x}+\frac{8}{x^2}\right)$$

$$D=x^5\left[\sqrt{\left(1+\frac{3}{2}\right)^5}+\sqrt[3]{\left(1+\frac{3}{x}\right)^4}\cdot\sqrt{1-\frac{2}{x}}+\dots+\sqrt{\left(1-\frac{2}{x}\right)^5}\right]$$

$$\frac{N}{D}=\frac{\left(12-\frac{3}{x}+\frac{8}{x^2}\right)}{\sqrt{\left(1+\frac{3}{x}\right)^5}+\sqrt{\left(1+\frac{3}{x}\right)^4}\cdot\sqrt{1-\frac{2}{x}}+\dots+\sqrt{\left(1-\frac{2}{x}\right)^5}}$$

$$\lim_{x\to+\infty}\left(\sqrt[3]{x^3+3x^2}-\sqrt{x^2-2x}\right)$$

$$=\lim_{x\to+\infty}\frac{12-\frac{3}{x}+\frac{8}{x^2}}{\sqrt{\left(1+\frac{3}{x}\right)^5}+\sqrt{\left(1+\frac{3}{x}\right)^4}\cdot\sqrt{1-\frac{2}{x}}+\quad+\sqrt{\left(1-\frac{2}{x}\right)^5}}$$

$$=\frac{12}{6}=2$$

$$\boxed{\lim_{x\to+\infty}\left(\sqrt[3]{x^3+3x^2}-\sqrt{x^2-2x}\right)=2}$$

លំហាត់ 27

$$\lim_{x\to\pm\infty}\left(\sqrt{x^2+4x-7}-\sqrt{x^2-4}\right)$$ លីមីតនេះមានរាងមិនកំណត់ $\infty-\infty$

គេអាចប្រើកន្សោមឆ្លាស់ដើម្បីគណនា

$$\lim_{x\to\pm\infty}\left(\sqrt{x^2+4x+7}-\sqrt{x^2-4}\right)=\lim_{x\to\pm\infty}\frac{4x+11}{\sqrt{x^2+4x+7}+\sqrt{x^2-4}}$$

$$=\lim_{x\to\pm\infty}\frac{4x+11}{|x|\sqrt{1+\frac{4}{x}+\frac{7}{x^2}}+|x|\sqrt{1-\frac{4}{x^2}}}$$

$$=\lim_{x\to\pm\infty}\frac{4x+11}{|x|\left(\sqrt{1+\frac{4}{x}+\frac{7}{x^2}}+\sqrt{1-\frac{4}{x^2}}\right)}$$

* បើ $x\to+\infty$ នោះ $|x|=x$

បើ $x\to-\infty$ នោះ $|x|=-x$ ដូចនេះគេបាន

$$\lim_{x\to\pm\infty}\left(\sqrt{x^2+4x+7}-\sqrt{x^2-4}\right)=\lim_{x\to\pm\infty}\frac{4x+11}{|x|\left(\sqrt{1+\frac{4}{x}+\frac{7}{x^2}}+\sqrt{1-\frac{4}{x^2}}\right)}$$

$$=\lim_{x\to\pm\infty}\frac{4x+11}{\pm x\left(\sqrt{1+\frac{4}{x}+\frac{7}{x^2}}+\sqrt{1-\frac{4}{x^2}}\right)}$$

$$=\lim_{x\to\pm\infty}\frac{4+\frac{11}{x}}{\pm\left(\sqrt{1+\frac{4}{x}+\frac{7}{x^2}}+\sqrt{1-\frac{4}{x^2}}\right)}$$

$$=\frac{4}{\pm 2}=\pm 2$$

$$\boxed{\lim_{x\to\pm\infty}\left(\sqrt{x^2+4x+7}-\sqrt{x^2-4}\right)=\pm 2}$$

28. $\lim_{x\to\infty}\left(\sqrt[3]{x^3+x+1}-\sqrt[3]{x^3-x-1}\right)$ មានរាងមិនកំណត់ $\infty-\infty$

$$\sqrt[3]{x^3+x+1}-\sqrt[3]{x^3-x-1}=\frac{x^3+x+1-x^3+x+1}{\sqrt[3]{(x^3+x+1)^2}+\sqrt[3]{x^3+x+1}\cdot\sqrt[3]{x^3-x-1}+\sqrt[3]{(x^3-x-1)^2}}$$

$$=\frac{2x+2}{\sqrt[3]{\left[x^3\left(1+\frac{1}{x^2}+\frac{1}{x^3}\right)\right]^2}+\sqrt[3]{x^3\left(1+\frac{1}{x^2}+\frac{1}{x^3}\right)}\cdot\sqrt[3]{x^3\left(1-\frac{1}{x^2}-\frac{1}{x^3}\right)}+\dots}$$

$$=\frac{x\left(2+\frac{2}{x}\right)}{x^2\sqrt[3]{1+\frac{1}{x^2}+\frac{1}{x^3}}+x\sqrt[3]{1+\frac{1}{x^2}+\frac{1}{x^3}}\cdot x\sqrt[3]{1-\frac{1}{x^2}-\frac{1}{x^3}}+x^2\sqrt[3]{1-\frac{1}{x^2}-\frac{1}{x^3}}}$$

$$=\frac{\left(2+\frac{2}{x}\right)}{x\left[\sqrt[3]{1+\frac{1}{x^2}+\frac{1}{x^3}}+\sqrt[3]{\left(1+\frac{1}{x^2}+\frac{1}{x^3}\right)\left(1-\frac{1}{x^2}-\frac{1}{x^3}\right)}+\sqrt[3]{1-\frac{1}{x^2}-\frac{1}{x^3}}\right]}$$

$x\neq 0$

$$\lim_{x\to\infty}\left(\sqrt[3]{x^3+x+1}-\sqrt[3]{x^3-x-1}\right)$$

$$=\lim_{x\to\infty}\frac{2+\frac{2}{x}}{x\left[\sqrt[3]{1+\frac{1}{x^2}+\frac{1}{x^3}}+\sqrt[3]{\left(1+\frac{1}{x^2}+\frac{1}{x^3}\right)\left(1-\frac{1}{x^2}-\frac{1}{x^3}\right)}+\sqrt[3]{1-\frac{1}{x^2}-\frac{1}{x^3}}\right]}=0$$

$$\boxed{\lim_{x\to\infty}\left(\sqrt[3]{x^3+x+1}-\sqrt[3]{x^3-x-1}\right)=0}$$

29. $\lim\limits_{x\to\infty}\left(\sqrt{x+1}-\sqrt{x}\right)$ មានរាងមិនកំណត់ $\infty-\infty$

$$\sqrt{x+1}-\sqrt{x}=\frac{\left(\sqrt{x+1}-\sqrt{x}\right)\left(\sqrt{x+1}+\sqrt{x}\right)}{\sqrt{x+1}+\sqrt{x}}$$

$$=\frac{x+1-x}{\sqrt{x+1}+\sqrt{x}}=\frac{1}{\sqrt{x+1}+\sqrt{x}}$$

$$\lim_{x\to\infty}\left(\sqrt{x+1}-\sqrt{x}\right)=\lim_{x\to\infty}\frac{1}{\sqrt{x+1}+\sqrt{x}}=0$$

$$\boxed{\lim_{x\to\infty}\left(\sqrt{x+1}-\sqrt{x}\right)=0}$$

30. $\lim\limits_{x\to\infty}\left(\sqrt[3]{x^3+x+1}-\sqrt{x^2-x-1}\right)$ មានរាងមិនកំណត់ $\infty-\infty$

$$\sqrt[3]{x^3+x+1}-\sqrt{x^2-x-1}=\frac{(x^3+x+1)^2-(x^2-x-1)^3}{\sqrt[3]{(x^3+x+1)^5}+\sqrt[3]{(x^3+x+1)^4}\cdot\sqrt{x^2-x-1}+\dots+\sqrt{(x^2-x-1)^5}}$$

$$N=x^6+x^2+1+2x^4+2x^3+2x-\left[(x^2-x)^3-3(x^2-x)^2+3(x^2-x)-1\right]$$

$$=x^6+x^2+1+2x^4+2x^3+2x-\left[x^6-3x^5+3x^4-x^3-3x^4+6x^3-3x^2+3x^2-3x-1\right]$$

$$=3x^5+2x^4-3x^3+x^2-5x+2$$

$$=x^5\left(3+\frac{2}{x}-\frac{3}{x^2}+\frac{1}{x^3}+\frac{5}{x^4}+\frac{2}{x^5}\right)$$

$$D=\sqrt{\left[x\left(1+\frac{1}{x^2}+\frac{1}{x^3}\right)\right]^5}+\sqrt[3]{\left[x^3\left(1+\frac{1}{x^2}+\frac{1}{x^3}\right)\right]^4}\cdot\sqrt{x^2\left(1-\frac{1}{x}-\frac{1}{x^2}\right)}+\dots+\sqrt{\left[x^2\left(1-\frac{1}{x}-\frac{1}{x^2}\right)\right]^5}$$

$$=x^5\sqrt[3]{\left(1+\frac{1}{x^2}+\frac{1}{x^3}\right)^3}+x^4\sqrt{\left(1+\frac{1}{x^2}+\frac{1}{x^3}\right)^4}\cdot x\sqrt{1-\frac{1}{x}-\frac{1}{x^2}}+\dots+x^5\sqrt{\left(1-\frac{1}{x}-\frac{1}{x^2}\right)^5}$$

$$\frac{N}{D}=\frac{\left(3+\frac{2}{x}-\frac{3}{x^2}+\frac{1}{x^3}+\frac{5}{x^4}+\frac{2}{x^5}\right)}{\sqrt{\left(1+\frac{1}{x^2}+\frac{1}{x^3}\right)^3}+\sqrt{\left(1+\frac{1}{x^2}+\frac{1}{x^3}\right)^4}\cdot\sqrt{1-\frac{1}{x}-\frac{1}{x^2}}+\dots+\sqrt{\left(1-\frac{1}{x}-\frac{1}{x^2}\right)^5}}$$

$(x\neq 0)$

$$\lim_{x\to\infty}\frac{N}{D}=\frac{3}{6}=\frac{1}{2}$$

$$\boxed{\lim_{x\to\infty}\left(\sqrt[3]{x^3+x+1}-\sqrt{x^2-x-1}\right)=\frac{1}{2}}$$

ង. ការរកលីមីតតាមព្រាយ (លីមីតត្រីកោណមាត្រ)

រំលឹក

$$\boxed{\lim_{x\to 0}\frac{\sin x}{x}=1}$$

31. $\lim\limits_{x\to 0}\frac{1-\cos x}{x^2}$, 32. $\lim\limits_{x\to 0}\frac{\operatorname{tg}x}{x}$, 33. $\lim\limits_{x\to 0}\frac{1-\cos x}{x}$

34. $\lim\limits_{x\to 0}\frac{\sin 2x}{\operatorname{tg}3x}$ 35. $\lim\limits_{x\to a}\frac{\operatorname{tg}x-\operatorname{tg}a}{x-a}$ 36. $\lim\limits_{x\to 0}\frac{\operatorname{tg}x-\sin x}{x^3}$

37. $\lim\limits_{x\to\frac{\pi}{6}}\frac{2\sin x-1}{1-2\cos 2x}$ 38 $\lim\limits_{x\to 0}\frac{1-\cos x}{2\operatorname{tg}^2 x}$ 39 $\lim\limits_{x\to 0}\frac{2\sin x-\sin 2x}{x^3}$

40 $\lim\limits_{x\to 0}\frac{1-\cos x}{\sin^2 x}$

សំរាយបញ្ជាក់

31. $\lim\limits_{x\to 0} \dfrac{1-\cos x}{x^2}$

$1-\cos x = 2\sin^2 \dfrac{x}{2}$

$$\lim_{x\to 0}\frac{1-\cos x}{x^2} = \lim_{x\to 0}\frac{2\sin^2 \frac{x}{2}}{x^2} = \lim_{x\to 0}\frac{\left(\sin \frac{x}{2}\right)^2}{\left(\frac{x}{2}\right)^2}\times\frac{1}{2} = \frac{1}{2}$$

$$\boxed{\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}}$$

32. $\lim\limits_{x\to 0}\dfrac{\operatorname{tg} x}{x}$

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{x} = \lim_{x\to 0}\frac{\sin x}{x\cos x} = \lim_{x\to 0}\frac{\sin x}{x}\times\lim_{x\to 0}\frac{1}{\cos x} = 1\times 1 = 1$$

$$\boxed{\lim_{x\to 0}\frac{\operatorname{tg} x}{x} = 1}$$

33. $\lim\limits_{x\to 0}\dfrac{1-\cos x}{x}$

$\cos 2x = 1-2\sin^2 x \Rightarrow 2\sin^2 x = 1-\cos 2x$

$2\sin^2\dfrac{x}{2} = 1-\cos x$

$$\frac{1-\cos x}{x} = \frac{2\sin^2 \frac{x}{2}}{x} = \frac{\sin^2 \frac{x}{2}}{\frac{x}{2}} = \frac{\sin\frac{x}{2}}{\frac{x}{2}}\times\sin\frac{x}{2}$$

$$\lim_{x\to 0}\frac{1-\cos x}{x} = \lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\cdot\lim_{x\to 0}\sin\frac{x}{2} = 1\times 0 = 0$$

$$\boxed{\lim_{x\to 0}\frac{1-\cos x}{x} = 0}$$

34. $\lim\limits_{x\to 0}\dfrac{\sin 2x}{\operatorname{tg} 3x}$

$$\frac{\sin 2x}{\operatorname{tg} 3x} = \frac{\sin 2x}{\frac{\sin 3x}{\cos 3x}} = \frac{\sin 2x}{2x}\times 2x\times\frac{1}{\sin 3x}\times\cos 3x$$

$$= \frac{\sin 2x}{2x}\times 2x\times\frac{1}{3x}\times\frac{1}{\frac{\sin 3x}{3x}}\times\cos 3x$$

$$= \frac{2}{3}\cdot\frac{\sin 2x}{2x}\times\frac{1}{\frac{\sin 3x}{3x}}\times\cos 3x$$

$$\lim_{x\to 0}\frac{\sin 2x}{\operatorname{tg} 3x} = \frac{2}{3}\lim_{x\to 0}\frac{\sin 2x}{2x}\cdot\lim_{x\to 0}\frac{1}{\frac{\sin 3x}{3x}}\lim_{x\to 0}\cos 3x$$

$$= \frac{2}{3}\cdot 1\cdot\frac{1}{1}\cdot 1 = \frac{2}{3}$$

$$\boxed{\lim_{x\to 0}\frac{\sin 2x}{\operatorname{tg} 3x} = \frac{2}{3}}$$

35. $\lim\limits_{x\to a}\dfrac{\operatorname{tg} x-\operatorname{tg} a}{x-a}$

$$\frac{\operatorname{tg} x-\operatorname{tg} a}{x-a} = \frac{1}{x-a}\left(\frac{\sin x}{\cos x}-\frac{\sin a}{\cos a}\right)$$

$$= \frac{1}{x-a}\left(\frac{\sin x\cos a-\sin a\cos x}{\cos x\cos a}\right)$$

$$= \frac{1}{x-a}\left(\frac{\sin(x-a)}{\cos x \cos a}\right)$$

$$= \frac{\sin(x-a)}{x-a} \cdot \frac{1}{\cos x \cos a}$$

$$\lim_{x\to a}\frac{tg x - tg a}{x-a} = \lim_{x\to a}\frac{\sin(x-a)}{x-a} \quad \lim_{x\to a}\frac{1}{\cos x \cos a} = \frac{1}{\cos^2 a}$$

$$\boxed{\lim_{x\to a}\frac{tg x - tg a}{x-a} = \frac{1}{\cos^2 a}}$$

36 $\lim_{x\to 0}\frac{tg x - \sin x}{x^3}$

$$\frac{tg x - \sin x}{x^3} = \frac{\frac{\sin x}{\cos x} - \sin x}{x^3} = \frac{\sin x}{x} \cdot \left(\frac{1-\cos x}{x^2 \cos x}\right)$$

$$= \frac{\sin x}{x} \cdot \frac{2\sin^2 \frac{x}{2}}{x^2} \cdot \frac{1}{\cos x}$$

$$= \frac{\sin x}{x} \cdot \frac{2}{4} \cdot \left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 \cdot \frac{1}{\cos x}$$

$$\lim_{x\to 0}\frac{tg x - \sin x}{x^3} = \lim_{x\to 0}\frac{\sin x}{x} \cdot \frac{1}{2} \cdot \lim_{x\to 0}\left(\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2 \cdot \lim_{x\to 0}\frac{1}{\cos x}$$

$$= 1 \cdot \frac{1}{2} \cdot 1^2 \cdot \frac{1}{1} = \frac{1}{2}$$

$$\boxed{\lim_{x\to 0}\frac{tg x - \sin x}{x^3} = \frac{1}{2}}$$

37. $\lim_{x\to \frac{\pi}{6}}\frac{2\sin x - 1}{1 - 2\cos 2x}$

$$\frac{2\sin x - 1}{1-2\cos 2x} = \frac{2\sin x - 1}{1-2(1-2\sin^2 x)} = \frac{2\sin x - 1}{4\sin^2 x - 1}$$

$$= \frac{2\sin x - 1}{(2\sin x + 1)(2\sin x - 1)} = \frac{1}{2\sin x + 1}$$

$$\lim_{x\to \frac{\pi}{6}}\frac{2\sin x - 1}{1-2\cos 2x} = \lim_{x\to \frac{\pi}{6}}\frac{1}{2\sin x + 1} = \frac{1}{2\cdot\frac{1}{2} + 1} = \frac{1}{2}$$

$$\boxed{\lim_{x\to \frac{\pi}{6}}\frac{2\sin x - 1}{1-2\cos 2x} = \frac{1}{2}}$$

38. $\lim_{x\to 0}\frac{1-\cos x}{2 tg^2 x}$

$$\frac{1-\cos x}{2 tg^2 x} = \frac{1-\cos x}{x^2} \; \frac{1}{2}\left(\frac{x}{tg x}\right)^2$$

$$\lim_{x\to 0}\frac{1-\cos x}{2 tg^2 x} = \lim_{x\to 0}\frac{1-\cos x}{x^2} \cdot \frac{1}{2} \cdot \lim_{x\to 0}\left(\frac{x}{tg x}\right)^2 = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

$$\boxed{\lim_{x\to 0}\frac{1-\cos x}{2 tg^2 x} = \frac{1}{4}}$$ (ព្រោះ: $\lim_{x\to 0}\frac{1-\cos x}{x^2} = \frac{1}{2}$)

39 $\lim_{x\to 0}\frac{2\sin x - \sin 2x}{x^3}$

$$\frac{2\sin x - \sin 2x}{x^3} = \frac{2\sin x - 2\sin x \cos x}{x^3} = \frac{2\sin x(1-\cos x)}{x^3}$$

$$=\frac{2\sin x}{x}\quad\frac{1-\cos x}{x^2}$$

$$\lim_{x\to 0}\frac{2\sin x-\sin 2x}{x^3}=2\lim_{x\to 0}\frac{\sin x}{x}\cdot\lim_{x\to 0}\frac{1-\cos x}{x^2}=2\cdot\frac{1}{2}=1$$

$$\boxed{\lim_{x\to 0}\frac{2\sin x-\sin 2x}{x^3}=1}$$

40 $\lim\limits_{x\to 0}\dfrac{1-\cos x}{\sin^2 x}$

$$\frac{1-\cos x}{\sin^2 x}=\frac{2\sin^2\frac{x}{2}}{\left(2\sin\frac{x}{2}\cos\frac{x}{2}\right)^2}=\frac{1}{2}\cdot\frac{1}{\cos^2\frac{x}{2}}$$

$$\lim_{x\to 0}\frac{1-\cos x}{\sin^2 x}=\frac{1}{2}\lim_{x\to 0}\frac{1}{\cos^2\frac{x}{2}}=\frac{1}{2}$$

$$\boxed{\lim_{x\to 0}\frac{1-\cos x}{\sin^2 x}=\frac{1}{2}}$$

ចូររកលីមីតខាងក្រោម

ទ្រឹស្តីបទ

$$\boxed{\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=e}\qquad e\approx 2,718$$

បើ $\alpha=\frac{1}{x}$, $\alpha\to 0$ $\quad\boxed{\lim\limits_{\alpha\to 0}(1+\alpha)^{\frac{1}{\alpha}}=e}$

41. $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+5}$, 42 $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{3x}$

43 $\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^{x}$, 44 $\lim\limits_{x\to\infty}\left(\frac{x}{1+x}\right)^{x}$

45 $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{\frac{x+1}{x}}$, 46 $\lim\limits_{x\to\infty}\left(\frac{x^2+1}{x^2-1}\right)^{x^2}$

47 $\lim\limits_{x\to\pm\infty}\left(\frac{2x+1}{x-1}\right)^{x}$, 48 $\lim\limits_{x\to\infty}\left(\frac{x+1}{x-2}\right)^{2x-1}$

កំណែលំហាត់

41. $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+5}$

$$\left(1+\frac{1}{n}\right)^{n+5}=\left(1+\frac{1}{n}\right)^{n}\cdot\left(1+\frac{1}{n}\right)^{5}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+5}=\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n}\cdot\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{5}=e\cdot 1^5=e$$

$$\boxed{\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+5}=e}$$

42 $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{3x}$

$$\left(1+\frac{1}{x}\right)^{3x}=\left(1+\frac{1}{x}\right)^{x}\cdot\left(1+\frac{1}{x}\right)^{x}\cdot\left(1+\frac{1}{x}\right)^{x}$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{3x}=\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\cdot\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}\cdot\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{x}$$

$$=e\cdot e\cdot e=e^3$$

$$\boxed{\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{3x}=e^3}$$

43 . $\lim\limits_{x\to\infty}\left(1+\frac{2}{x}\right)^{x}$

តាង $\frac{1}{a}=\frac{2}{x}\Rightarrow x=2a$, $x\to\infty$, $a\to\infty$

$$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{x}=\lim_{a\to\infty}\left(1+\frac{1}{a}\right)^{2a}=\left[\lim_{a\to\infty}\left(1+\frac{1}{a}\right)^{a}\right]^2=e^2$$

$$\boxed{\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{x}=e^2}$$

44 $\lim\limits_{x\to\infty}\left(\frac{x}{1+x}\right)^{x}$

$$\lim_{x\to\infty}\left(\frac{x}{1+x}\right)^{x}=\lim_{x\to\infty}\left(1-\frac{1}{1+x}\right)^{x}=\lim_{x\to\infty}\left[\left(1-\frac{1}{1+x}\right)^{-(1+x)}\right]^{-\frac{x}{x+1}}$$

$$=e^{\lim\limits_{x\to\infty}\frac{-x}{x+1}}=e^{-1}=\frac{1}{e}$$

$$\boxed{\lim_{x\to\infty}\left(\frac{x}{x+1}\right)^{x}=\frac{1}{e}}$$

45. $\lim\limits_{x\to\infty}\left(1+\frac{1}{x}\right)^{\frac{x+1}{x}}$

$$=\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{\frac{x(x+1)}{x^2}}=e^{\lim\limits_{x\to\infty}\frac{x+1}{x^2}}$$

$$x\to\infty\Rightarrow\lim_{x\to\infty}\frac{x+1}{x^2}=\lim_{x\to\infty}\frac{x\left(1+\frac{1}{x}\right)}{x^2}=0$$

$$\boxed{\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^{\frac{x+1}{x}}=e^0=1}$$

46 . $\lim\limits_{x\to\infty}\left(\frac{x^2+1}{x^2-1}\right)^{x^2}$

$$= \lim_{x\to\infty}\left(1+\frac{2}{x^2-1}\right)^{x^2} \quad , \; x\to\infty \Rightarrow \frac{2}{x^2-1} = 0$$

តាង $\alpha = \dfrac{2}{x^2-1} \Rightarrow x^2 = \dfrac{2+\alpha}{\alpha}$

$$\lim_{x\to\infty}\left(\frac{x^2+1}{x^2-1}\right)^{x^2} = \lim_{\alpha\to 0}(1+\alpha)^{\frac{2+\alpha}{\alpha}} = \lim_{\alpha\to 0}\left[(1+\alpha)^{\frac{1}{\alpha}}\right]^{(2+\alpha)}$$

$$= e^{\lim\limits_{\alpha\to 0}(2+\alpha)} = e^2$$

$$\boxed{\lim_{x\to\infty}\left(\frac{x^2+1}{x^2-1}\right)^{x^2} = e^2}$$

47 $\displaystyle\lim_{x\to\pm\infty}\left(\frac{2x+1}{x-1}\right)^{x}$

$$\left(\frac{2x+1}{x-1}\right)^{x} = \frac{x-1+x+2}{x-1} = \left(1+\frac{x+2}{x-1}\right)^{x}$$

$x\to+\infty$ ដោយ: $\dfrac{x+2}{x-1}\to 1$

$$\lim_{x\to+\infty}\left(\frac{2x+1}{x-1}\right)^{x} = 2^{\infty} = \infty$$

$x\to-\infty$ ដោយ: $\dfrac{x+2}{x-1}\to 1$

$$\lim_{x\to-\infty}\left(\frac{2x+1}{x-1}\right)^{x} = 2^{-\infty} = \frac{1}{2^{\infty}} = \frac{1}{\infty} = 0$$

$$\boxed{\lim_{x\to+\infty}\left(\frac{2x+1}{x-1}\right)^{x} = \infty \; ; \; \lim_{x\to-\infty}\left(\frac{2x+1}{x-1}\right)^{x} = 0}$$

លំហាត់ 48

$$\lim_{x\to\infty}\left(\frac{x+1}{x-2}\right)^{2x-1} = \lim_{x\to\infty}\left(1+\frac{3}{x-2}\right)^{2x-1} = \lim_{x\to\infty}\left[\left(1+\frac{3}{x-2}\right)^{\frac{x-2}{3}}\right]^{\frac{3(2x-1)}{x-2}}$$

$$= e^{\lim\limits_{x\to\infty}\frac{3(2x-1)}{x-2}} = e^6$$

ព្រោះ: $\displaystyle\lim_{x\to\infty}\frac{3(2x-1)}{x-2} = 6$

ដូចនេះ: $$\boxed{\lim_{x\to\infty}\left(\frac{x+1}{x-2}\right)^{2x-1} = e^6}$$

លំហាត់ទូទៅលើលីមីត

ច្បាស់និងការព្រាង

49 គេឲ្យអនុគមន៍ $f(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n$

គណនា $\lim\limits_{x\to\infty} f(x)$

$$f(x) = a_0x^n + a_1x^{n-1} + \dots + a_{n-1}x + a_n$$

$$= a_0x^n\left[1 + \frac{a_1\,x^{n-1}}{a_0\,x^n} + \dots + \frac{a_{n-1}\cdot x}{a_0\,x^n} + \frac{a_n}{a_0\,x^n}\right]$$

$$= a_0x^n\left[1 + \frac{a_1}{a_0x} + \dots + \frac{a_{n-1}}{a_0x^{n-1}} + \frac{a_n}{a_0\cdot x^n}\right]$$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} a_0x^n\left[1 + \frac{a_1}{a_0x} + \dots + \frac{a_{n-1}}{a_0x^{n-1}} + \frac{a_n}{a_0x^n}\right]$$

$$= \lim_{x\to\infty} a_0x^n \cdot \lim_{x\to\infty}\left[1 + \frac{a_1}{a_0x} + \dots + \frac{a_{n-1}}{a_0x^{n-1}} + \frac{a_n}{a_0x^n}\right]$$

តែ $\lim\limits_{x\to\infty}\left[1 + \frac{a_1}{a_0x_n} + \dots + \frac{a_{n-1}}{a_0x^{n-1}} + \frac{a_n}{a_0x^n}\right]$

$$= \lim_{x\to\infty} 1 + \lim_{x\to\infty}\frac{a_1}{a_0x_n} + \dots + \lim_{x\to\infty}\frac{a_{n-1}}{a_0x^{n-1}} + \lim_{x\to\infty}\frac{a_n}{a_0x^n}$$

$$= 1 + 0 + \dots + 0 + 0$$

ដោយ $\lim\limits_{x\to\infty} a_0x^n = \infty$

នោះ $\lim\limits_{x\to\infty} a_0x^n\left[1 + \frac{a_1}{a_0x} + \dots + \frac{a_{n-1}}{a_0x^{n-1}} + \frac{a_n}{a_0x^n}\right] = \infty$

$$\boxed{\lim_{x\to\infty} f(x) = \infty}$$

50 គណនា $\lim\limits_{x\to a}\frac{\sqrt[m]{x} - \sqrt[m]{a}}{x - a}$

$$\frac{\sqrt[m]{x} - \sqrt[m]{a}}{x - a} = \frac{\sqrt[m]{x} - \sqrt[m]{a}}{(\sqrt[m]{x})^m - (\sqrt[m]{a})^m}$$

$$= \frac{\sqrt[m]{x} - \sqrt[m]{a}}{(\sqrt[m]{x} - \sqrt[m]{a})\left[(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2}\sqrt[m]{a} + \dots + (\sqrt[m]{a})^{m-1}\right]}$$

$$\lim_{x\to a}\frac{\sqrt[m]{x} - \sqrt[m]{a}}{x - a} = \lim_{x\to a}\frac{\sqrt[m]{x} - \sqrt[m]{a}}{(\sqrt[m]{x} - \sqrt[m]{a})\left[(\sqrt[m]{x})^{m-1} + (\sqrt[m]{x})^{m-2}\sqrt[m]{a} + \dots + (\sqrt[m]{a})^{m-1}\right]}$$

$$= \frac{1}{m(\sqrt[m]{a})^{m-1}} = \frac{(\sqrt[m]{a})}{ma}$$

ដូច្នេះលទ្ធផល

$$\boxed{\lim_{x\to a}\frac{\sqrt[m]{a} - \sqrt[m]{a}}{m - a} = \frac{\sqrt[m]{a}}{ma}}$$

51 គណនា $\lim\limits_{n\to\infty}\frac{3n + 1}{n^2 - 2n + 3}$

$$\frac{3n+1}{n^2-2n+3}=\frac{n\left(3+\frac{1}{n}\right)}{n^2\left(1-\frac{2}{n}+\frac{3}{n^2}\right)}=\frac{3+\frac{1}{n}}{n\left(1-\frac{2}{n}+\frac{3}{n^2}\right)}\quad (n\neq 0)$$

$$\lim_{n\to\infty}\frac{3n+1}{n^2-2n+3}=\lim_{n\to\infty}\frac{3+\frac{1}{n}}{n\left(1-\frac{2}{n}+\frac{3}{n^2}\right)}=\frac{3}{\infty}=0$$

$$\boxed{\lim_{n\to\infty}\frac{3n+1}{n^2-2n+3}=0}$$

52 គណនា $\lim\limits_{n\to\infty}\dfrac{n(2n-1)(3n-2)}{(2n+1)^3}$

$$\frac{n(2n-1)(3n-2)}{(2n+1)^3}=\frac{n^3\left[\left(\frac{2n-1}{n}\right)\left(\frac{3n-2}{n}\right)\right]}{n^3\left(\frac{2n+1}{n}\right)^3}$$

$$=\frac{\left(2-\frac{1}{n}\right)\left(3-\frac{2}{n}\right)}{\left(2+\frac{1}{n}\right)^3}\quad (n\neq 0)$$

$$\lim_{n\to\infty}\frac{n(2n-1)(3n-2)}{(2n+1)^3}=\lim_{n\to\infty}\frac{\left(2-\frac{1}{n}\right)\left(3-\frac{2}{n}\right)}{\left(2+\frac{1}{n}\right)^3}=\frac{6}{8}=\frac{3}{4}$$

$$\boxed{\lim_{n\to\infty}\frac{n(2n-1)(3n-2)}{(2n+1)^3}=\frac{3}{4}}$$

53. គណនា $\lim\limits_{n\to\infty}\dfrac{(2n-1)^2(n+3)^3}{2(n+1)^5}$

$$\frac{(2n-1)^2(n+3)^3}{2(n+1)^5}=\frac{n^5\left(2-\frac{1}{n}\right)^2\left(1+\frac{3}{n}\right)^3}{2n^5\left(1+\frac{1}{n}\right)^5}$$

$$=\frac{\left(2-\frac{1}{n}\right)^2\left(1+\frac{3}{n}\right)^3}{2\left(1+\frac{1}{n}\right)^5}\quad (n\neq 0)$$

$$\lim_{n\to\infty}\frac{(2n-1)^2(m+3)^3}{2(n+1)^5}=\lim_{n\to\infty}\frac{\left(2-\frac{1}{n}\right)^2\left(1+\frac{3}{n}\right)^3}{2\left(1+\frac{1}{n}\right)^5}=\frac{4}{2}=2$$

$$\boxed{\lim_{n\to\infty}\frac{(2n-1)^2(n+3)^3}{2(n+1)^5}=2}$$

54 គណនា $\lim\limits_{n\to\infty}\dfrac{9n^2-7n+1}{4n^3+n+3}$

$$\frac{9n^2-7n+1}{4n^3+n+3}=\frac{n^2\left(9-\frac{7}{n}+\frac{1}{n^2}\right)}{n^3\left(4+\frac{1}{n^2}+\frac{3}{n^3}\right)}=\frac{9-\frac{7}{n}+\frac{1}{n^2}}{n\left(4+\frac{1}{n^2}+\frac{3}{n^3}\right)}\quad n\neq 0$$

$$\lim_{n\to\infty}\frac{9n^2-7n+1}{4n^3+n+3}=\lim_{n\to\infty}\frac{9-\frac{7}{n}+\frac{1}{n^2}}{n\left(4+\frac{1}{n^2}+\frac{3}{n^3}\right)}=\frac{9}{\infty}=0$$

$$\boxed{\lim_{n\to\infty}\frac{9n^2-7n+1}{4n^3+n+3}=0}$$

55 គណនា $\lim\limits_{n\to\infty}\dfrac{n\sqrt{n}-3}{5n^2+1}$

$$\frac{n\sqrt{n}-3}{5n^2+1} = \frac{n^2\left(\sqrt{\frac{1}{n}}-\frac{3}{n^2}\right)}{n^2\left(5+\frac{1}{n^2}\right)} = \frac{\sqrt{\frac{1}{n}}-\frac{3}{n^2}}{5+\frac{1}{n^2}} \qquad (n\neq 0)$$

$$\lim_{n\to\infty}\frac{n\sqrt{n}-3}{5n^2+1} = \lim_{n\to\infty}\frac{\sqrt{\frac{1}{n}}-\frac{3}{n^2}}{5+\frac{1}{n^2}} = 0$$

$$\boxed{\lim_{n\to\infty}\frac{n\sqrt{n}-3}{5n^2+1} = 0}$$

56 គណនា $\lim\limits_{n\to\infty}\dfrac{1000n^3+3n^2}{0{,}001n^4-100n^3+1}$

$$\frac{1000n^3+3n^2}{0{,}001n^4-100n^3+1} = \frac{n^3\left(1000+\frac{3}{n}\right)}{n^4\left(0{,}001-\frac{100}{n}+\frac{1}{n^4}\right)} = \frac{1000}{n\left(0{,}001-\frac{100}{n}+\frac{1}{n^4}\right)}$$

$(n\neq 0)$

$$\lim_{n\to\infty}\frac{1000n^3+3n^2}{0{,}001n^4-100n^3+1} = \lim_{n\to\infty}\frac{1000+\frac{3}{n}}{n\left(0{,}001-\frac{100}{n}+\frac{1}{n^4}\right)} = 0$$

$$\boxed{\lim_{n\to\infty}\frac{1000n^3+3n^2}{0{,}001n^4-100n^3+1} = 0}$$

57. គណនា $\lim\limits_{n\to\infty}\dfrac{\sqrt{2n-1}-\sqrt{n}}{\sqrt{3n+1}}$

$$\frac{\sqrt{2n-1}-\sqrt{n}}{\sqrt{3n+1}} = \frac{\sqrt{n}\left(\sqrt{2-\frac{1}{n}}-1\right)}{\sqrt{n}\left(\sqrt{3+\frac{1}{n}}\right)} = \frac{\sqrt{2-\frac{1}{n}}-1}{\sqrt{3+\frac{1}{n}}} \quad (\sqrt{n}\neq 0)$$

$$\lim_{n\to\infty}\frac{\sqrt{2n-1}-\sqrt{n}}{\sqrt{3n+1}} = \lim_{n\to\infty}\frac{\sqrt{2-\frac{1}{n}}-1}{\sqrt{3+\frac{1}{n}}} = \frac{\sqrt{2}-1}{\sqrt{3}}$$

$$\boxed{\lim_{n\to\infty}\frac{\sqrt{2n-1}-\sqrt{n}}{\sqrt{3n+1}} = \frac{\sqrt{2}-1}{\sqrt{3}}}$$

58. គណនា $\lim\limits_{n\to\infty}\dfrac{1+\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^n}}{1+\frac{1}{3}+\frac{1}{9}+\cdots+\frac{1}{3^n}}$

$$\frac{1+\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^n}}{1+\frac{1}{3}+\frac{1}{9}+\cdots+\frac{1}{3^n}} = \frac{\dfrac{\left(\frac{1}{2}\right)^n-1}{-\frac{1}{2}}}{\dfrac{\left(\frac{1}{3}\right)^n-1}{-\frac{2}{3}}} = \frac{\left(\frac{1}{2}\right)^n-1}{\left(\frac{1}{3}\right)^n-1}\times\frac{4}{3}$$

$$\lim_{n\to\infty}\frac{1+\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^n}}{1+\frac{1}{3}+\frac{1}{9}+\cdots+\frac{1}{3^n}} = \lim_{n\to\infty}\frac{\frac{1}{2^n}-1}{\frac{1}{3^n}-1}\times\frac{4}{3} = \frac{4}{3}$$

$$\boxed{\lim_{n\to\infty}\frac{1+\frac{1}{2}+\frac{1}{4}+\cdots+\frac{1}{2^n}}{1+\frac{1}{3}+\frac{1}{9}+\cdots+\frac{1}{3^n}} = \frac{4}{3}}$$

59. គណនា $\lim\limits_{n\to\infty}\dfrac{1}{n^2}(1+2+3+\cdots+n)$

$$\frac{1}{n^2}(1+2+3+\cdots+n) = \frac{1}{n^2}\left[\frac{n}{2}(n+1)\right] = \frac{1}{n^2}\left(\frac{n^2}{2}+\frac{n}{2}\right)$$

$$= \frac{1}{2}+\frac{1}{2n}$$

$$\lim_{n\to\infty}\frac{1}{n^2}(1+2+3+\cdots+n) = \lim_{n\to\infty}\left(\frac{1}{2}+\frac{1}{2n}\right) = \frac{1}{2}$$

$$\boxed{\lim_{n\to\infty} \frac{1}{n^2}(1+2+3+\cdots+n)=\frac{1}{2}}$$

60. គណនា $\lim_{n\to\infty}\left(\frac{1+2+3+\cdots+n}{n+2}-\frac{n}{2}\right)$

$$\frac{1+2+3+\cdots+n}{n+2}-\frac{n}{2}=\frac{\frac{n}{2}(n+1)}{n+2}-\frac{n}{2}=\frac{\frac{n^2}{2}+\frac{n}{2}-\frac{n^2}{2}-n}{n+2}$$

$$=\frac{-\frac{n}{2}}{n+2}=\frac{-n\left(\frac{1}{2}\right)}{n\left(1+\frac{2}{n}\right)}=\frac{-\frac{1}{2}}{1+\frac{2}{n}}$$

$(n\neq 0)$

$$\lim_{n\to\infty}\left(\frac{1+2+3+\cdots+n}{n+2}-\frac{n}{2}\right)=\lim_{n\to\infty}\frac{-\frac{1}{2}}{1+\frac{2}{n}}=-\frac{1}{2}$$

$$\boxed{\lim_{n\to\infty}\left(\frac{1+2+3+\cdots+n}{n+2}-\frac{n}{2}\right)=-\frac{1}{2}}$$

61. គណនា $\lim_{n\to\infty}\left(\frac{1-2+3-4+\cdots-2n}{\sqrt{n^2+1}}\right)$

$$\frac{1-2+3-4+\cdots-2n}{\sqrt{n^2+1}}=\frac{1+3+5+\cdots+(2n-1)-(2+4+\cdots+2n)}{\sqrt{n^2+1}}$$

$$=\frac{n^2-n-n^2}{n\sqrt{1+\frac{1}{n^2}}}=\frac{-1}{\sqrt{1+\frac{1}{n^2}}}\quad (n\neq 0)$$

$$\lim_{n\to\infty}\left(\frac{1-2+3+4+\cdots-2n}{\sqrt{n^2+1}}\right)=\lim_{n\to\infty}\frac{-1}{\sqrt{1+\frac{1}{n^2}}}=-1$$

$$\boxed{\lim_{n\to\infty}\left(\frac{1-2+3-4+\cdots-2n}{\sqrt{n^2+1}}\right)=-1}$$

62. គណនា $\lim_{n\to\infty}\left(\frac{1}{1.2}+\frac{1}{2.3}+\cdots+\frac{1}{(n-1)n}\right)$

យើងដឹងថា $\frac{1}{1.2}=\left(1-\frac{1}{2}\right)$

$$\frac{1}{2.3}=\left(\frac{1}{2}-\frac{1}{3}\right)$$

$$\vdots$$

$$\frac{1}{(n-1)n}=\left(\frac{1}{n-1}-\frac{1}{n}\right)$$

$$S_n=1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\cdots+\frac{1}{n-1}-\frac{1}{n}$$

$$=1-\frac{1}{n}$$

$$\lim_{n\to\infty}\frac{1}{1.2}+\frac{1}{2.3}+\cdots+\frac{1}{(n-1)n}=\lim_{n\to\infty}\left(1-\frac{1}{n}\right)=1$$

$$\boxed{\lim_{n\to\infty}\frac{1}{1.2}+\frac{1}{2.3}+\cdots+\frac{1}{(n-1)n}=1}$$

63. គណនា $\lim_{n\to\infty}\left(\frac{1}{1.3}+\frac{1}{3.5}+\cdots+\frac{1}{(2n-1)(2n+1)}\right)$

យើងដឹងថា $\frac{1}{1.3}=\frac{1}{2}\left(1-\frac{1}{3}\right)$

$$\frac{1}{3.5} = \frac{1}{2}\left(1 - \frac{1}{3}\right)$$

$$\vdots \qquad\qquad \vdots$$

$$\frac{1}{(2n-1)(2n+1)} = \frac{1}{2}\left(\frac{1}{2n-1} - \frac{1}{2n+1}\right)$$

$$S_n = \frac{1}{2}\left(1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \dots + \frac{1}{2n-1} - \frac{1}{2n+1}\right)$$

$$= \frac{1}{2}\left(1 - \frac{1}{2n+1}\right)$$

$$\lim_{n\to\infty}\left(\frac{1}{1.3} + \frac{1}{3.5} + \dots + \frac{1}{(2n-1)(2n+1)}\right) = \frac{1}{2}\lim_{n\to\infty}\left(1 - \frac{1}{2n+1}\right) = \frac{1}{2}$$

$$\boxed{\lim_{n\to\infty} \frac{1}{1.3} + \frac{1}{3.5} + \dots + \frac{1}{(2n-1)(2n+1)} = \frac{1}{2}}$$

64. គណនា $\lim_{x\to 1} \frac{x + x^2 + \dots + x^n - n}{x-1} \quad (n \in \mathbb{N})$

$$\frac{x + x^2 + \dots + x^n - n}{x-1} = \frac{(x-1) + (x^2-1) + \dots + (x^n - 1)}{x-1}$$

$$= \frac{(x-1)\left[1 + (x+1) + (x^2+x+1) + \dots + (x^{n-1} + \dots + x + 1)\right]}{x-1}$$

$$= \left[1 + (x+1) + (x^2+x+1) + \dots + (x^{n-1} + \dots + x + 1)\right] \quad (x \neq 1)$$

$$\lim_{x\to 1}\frac{x + x^2 + \dots + x^n - n}{x-1} = \lim_{x\to 1}\left[1 + (x+1) + (x^2+x+1) + \dots + x^{n-1} + \dots + (x+1)\right]$$

$$= 1 + 2 + 3 + \dots + n = \frac{n}{2}(n+1)$$

$$\boxed{\lim_{x\to 1}\frac{x + x^2 + \dots + x^n - n}{x-1} = \frac{n}{2}(n+1)}$$

65. គណនា $\lim_{n\to\infty} \frac{n\sqrt{n^2+n} - 2n + 1}{2n^2 + 3n + 2}$

$$\frac{n\sqrt{n^2+n} - 2n + 1}{2n^2+3n+2} = \frac{n^2\left(\sqrt{1+\frac{1}{n}} - \frac{2}{n} + \frac{1}{n^2}\right)}{n^2\left(2 + \frac{3}{n} + \frac{2}{n^2}\right)}$$

$$= \frac{\sqrt{1+\frac{1}{n}} - \frac{2}{n} + \frac{1}{n^2}}{2 + \frac{3}{n} + \frac{2}{n^2}} \quad (n \neq 0)$$

$$\lim_{n\to\infty}\frac{n\sqrt{n^2+n} - 2n + 1}{2n^2+3n+2} = \lim_{n\to\infty}\frac{\sqrt{1+\frac{1}{n}} - \frac{2}{n} + \frac{1}{n^2}}{2 + \frac{3}{n} + \frac{2}{n^2}} = \frac{1}{2}$$

$$\boxed{\lim_{n\to\infty}\frac{n\sqrt{n^2+n} - 2n + 1}{2n^2+3n+2} = \frac{1}{2}}$$

66. គណនា $\lim_{n\to\infty} \frac{3\sqrt[3]{n^3+n+1} - 2\sqrt{n^2-1}}{3n+2}$

$$\frac{3\sqrt[3]{n^3+n+1} - 2\sqrt{n^2-1}}{3n+2} = \frac{3n\sqrt[3]{1 + \frac{1}{n^2} + \frac{1}{n^3}} - 2n\sqrt{1 - \frac{1}{n^2}}}{n\left(3 + \frac{2}{n}\right)}$$

$$= \frac{3\sqrt[3]{1+\frac{1}{n^2}+\frac{1}{n^3}} - 2\sqrt{1-\frac{1}{n^2}}}{3+\frac{2}{n}} \qquad n \neq 0$$

$$\lim_{n\to\infty} \frac{3\sqrt[3]{n^3+n+1} - 2\sqrt{n^2-1}}{3n+2} = \lim_{n\to\infty} \frac{3\sqrt[3]{1+\frac{1}{n^2}+\frac{1}{n^3}} - 2\sqrt{1-\frac{1}{n^2}}}{3+\frac{2}{n}}$$

$$= \frac{3-2}{3} = \frac{1}{3}$$

$$\boxed{\lim_{n\to\infty} \frac{3\sqrt[3]{n^3+n+1} - 2\sqrt{n^2-1}}{3n+2} = \frac{1}{3}}$$

67. គណនា $\displaystyle\lim_{n\to\infty} \frac{n\sqrt{n^3-n+3} - n^2\sqrt[3]{n}}{n^2\cdot\sqrt{n^2+1} - 1}$

$$\frac{n\sqrt{n^3-n+3} - n^2\sqrt[3]{n}}{n^2\sqrt{n^2+1} - 1} = \frac{n^3\left[\sqrt{\frac{1}{n}-\frac{1}{n^3}+\frac{3}{n^4}} - \sqrt[3]{\frac{1}{n^2}}\right]}{n^3\left[\sqrt{1+\frac{1}{n^2}} - \frac{1}{n^3}\right]}$$

$$= \frac{\sqrt{\frac{1}{n}-\frac{1}{n^3}+\frac{3}{n^4}} - \sqrt[3]{\frac{1}{n^2}}}{\sqrt{1+\frac{1}{n^2}} - \frac{1}{n^3}} \quad n \neq 0$$

$$\lim_{n\to\infty} \frac{n\sqrt{n^3-n+3} - n^2\sqrt[3]{n}}{n^2\sqrt{n^2+1} - 1} = \lim_{n\to\infty} \frac{\sqrt{\frac{1}{n}-\frac{1}{n^2}+\frac{3}{n^4}} - \sqrt[3]{\frac{1}{n^2}}}{\sqrt{1+\frac{1}{n^2}} - \frac{1}{n^3}} = 0$$

$$\boxed{\lim_{n\to\infty} \frac{n\sqrt{n^3-n+3} - n^2\sqrt[3]{n}}{n^2\cdot\sqrt{n^2+1} - 1} = 0}$$

68. គណនា $\displaystyle\lim_{n\to\infty} \frac{n^3\cdot\sqrt{n^2-1}}{n^2\sqrt[3]{n^2+n+1}}$

$$\frac{n^3\cdot\sqrt{n^2-1}}{n^2\sqrt[3]{n^2+n+1}} = \frac{n^4\sqrt{1-\frac{1}{n^2}}}{n^4\cdot\sqrt[3]{\frac{1}{n^4}+\frac{1}{n^5}+\frac{1}{n^6}}} = \frac{\sqrt{1-\frac{1}{n^2}}}{\sqrt{\frac{1}{n^4}+\frac{1}{n^5}+\frac{1}{n^6}}} \quad (n\neq 0)$$

$$\lim_{n\to\infty} \frac{n^3\sqrt{n^2-1}}{n^2\sqrt[3]{n^2+n+1}} = \lim_{n\to\infty} \frac{\sqrt{1-\frac{1}{n^2}}}{\sqrt{\frac{1}{n^4}+\frac{1}{n^5}+\frac{1}{n^6}}} = \frac{1}{0} = \infty$$

$$\boxed{\lim_{n\to\infty} \frac{n^3\cdot\sqrt{n^2-1}}{n^2\sqrt{n^2+n+1}} = \infty}$$

69. គណនា $\displaystyle\lim_{n\to\infty} n\left[\sqrt{n^2+1} - \sqrt{n^2-2}\right]$

$$n\left[\sqrt{n^2+1} - \sqrt{n^2-2}\right] = \frac{n\left[(n^2+1)-(n^2-2)\right]}{\sqrt{n^2+1}+\sqrt{n^2-2}}$$

$$= \frac{3n}{\sqrt{n^2+1}+\sqrt{n^2-2}} = \frac{3n}{n\left[\sqrt{1+\frac{1}{n^2}}+\sqrt{1-\frac{2}{n^2}}\right]} \quad (n\neq 0)$$

$$= \frac{3}{\sqrt{1+\frac{1}{n^2}}+\sqrt{1-\frac{2}{n^2}}}$$

$$\lim_{n\to\infty} n\left[\sqrt{n^2+1} - \sqrt{n^2-2}\right] = \lim_{n\to\infty} \frac{3}{\sqrt{1+\frac{1}{n^2}}+\sqrt{1-\frac{2}{n^2}}} = \frac{3}{2}$$

$$\boxed{\lim_{n\to\infty} n\left(\sqrt{n^2+1}-\sqrt{n^2-2}\right)=\frac{3}{2}}$$

70 . គណនា $\lim\limits_{n\to\infty} n\left[\sqrt[3]{n-n^3}+n\right]$

$$n\left[\sqrt[3]{n-n^3}+n\right]=\frac{n\left[(n-n^3)+n^3\right]}{\sqrt[3]{(n-n^3)^2}-n\sqrt[3]{n-n^3}+n^2}$$

$$=\frac{n^2}{n^2\left[\sqrt[3]{\left(\frac{1}{n^2}-1\right)^2}-\sqrt[3]{\frac{1}{n^2}-1}+1\right]}$$

$$=\frac{1}{\sqrt[3]{\left(\frac{1}{n^2}-1\right)^2}-\sqrt[3]{\frac{1}{n^2}-1}+1}\qquad (n\neq 0)$$

$$\lim_{n\to\infty} n\left[\sqrt[3]{n-n^3}+n\right]=\lim_{n\to\infty}\frac{1}{\sqrt[3]{\left(\frac{1}{n^2}-1\right)^2}-\sqrt[3]{\frac{1}{n^2}-1}+1}=\frac{1}{3}$$

$$\boxed{\lim_{n\to\infty} n\left[\sqrt[3]{n-n^3}+n\right]=\frac{1}{3}}$$

71. គណនា $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x-2}+\sqrt[3]{1-x+x^2}}{x^2-1}$

$$\frac{\sqrt[3]{x-2}+\sqrt[3]{1-x+x^2}}{x^2-1}=\frac{(x-2)+(1-x+x^2)}{(x^2-1)\left[\sqrt[3]{(x-2)^2}-\sqrt[3]{(x-2)(1-x+x^2)}+\sqrt[3]{(1-x+x^2)^2}\right]}$$

$$=\frac{x^2-1}{(x^2-1)\left[\sqrt[3]{(x-2)^2}-\sqrt[3]{(x-2)(1-x+x^2)}+\sqrt[3]{(1-x+x^2)^2}\right]}$$

$$=\frac{1}{\left[\sqrt[3]{(x-2)^2}-\sqrt[3]{(x-2)(1-x+x^2)}+\sqrt[3]{(1-x+x^2)^2}\right]}$$

$$(x\neq 1,\ x\neq 1)$$

$$\lim_{x\to 1}\frac{\sqrt[3]{x-2}+\sqrt[3]{1-x+x^2}}{x^2-1}=\lim_{x\to 1}\frac{1}{\sqrt[3]{(x-2)^2}-\sqrt[3]{(x-2)(1-x+x^2)}+\sqrt[3]{(1-x+x^2)^2}}=\frac{1}{3}$$

$$\boxed{\lim_{x\to 1}\frac{\sqrt[3]{x-2}+\sqrt[3]{1-x+x^2}}{x^2-1}=\frac{1}{3}}$$

គណនា $\lim\limits_{x\to 1}\dfrac{\sqrt[n]{x}-1}{\sqrt[m]{x}-1}$

ដោយដឹងថា $a^n-b^n=(a-b)(a^{n-1}+a^{n-2}b+\dots+ab^{n-2}+b^{n-1})$
គេបានថា

$$\lim_{x\to 1}\frac{\sqrt[n]{x}-1}{\sqrt[m]{x}-1}=\lim_{x\to 1}\frac{(x-1)\left(\sqrt[m]{x^{m-1}}+\sqrt[m]{x^{m-2}}+\dots+1\right)}{(x-1)\left(\sqrt[n]{x^{n-1}}+\sqrt[n]{x^{n-2}}+\dots+1\right)}$$

$$= \frac{\overbrace{1+1+\cdots+1}^{m\ \text{ដង}}}{\underbrace{1+1+\cdots+1}_{n\ \text{ដង}}} = \frac{m}{n}$$

សម្គាល់:

$$\boxed{\lim_{x\to 1} \frac{\sqrt[n]{x}-1}{\sqrt[m]{x}-1} = \frac{m}{n}}$$

73. គណនា $\lim\limits_{x\to 0} \dfrac{1-\sqrt[3]{1-x}}{3x}$

$$\frac{1-\sqrt[3]{1-x}}{3x} = \frac{1-(1-x)}{3x\left[1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}\right]}$$

$$= \frac{1}{3\left[1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}\right]} \qquad (x\neq 0)$$

$$\lim_{x\to 0} \frac{1-\sqrt[3]{1-x}}{3x} = \lim_{x\to 0} \frac{1}{3\left[1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}\right]} = \frac{1}{9}$$

$$\boxed{\lim_{x\to 0} \frac{1-\sqrt[3]{1-x}}{3x} = \frac{1}{9}}$$

74. គណនា $\lim\limits_{x\to -1} \dfrac{\sqrt[3]{x}+1}{\sqrt{x^2+3}-2}$

$$\frac{\sqrt[3]{x}+1}{\sqrt{x^2+3}-2} = \frac{(x+1)\left(\sqrt{x^2+3}+2\right)}{(x^2+3-4)\left(\sqrt[3]{x^2}-\sqrt[3]{x}+1\right)}$$

$$= \frac{(x+1)\left(\sqrt{x^2+3}+2\right)}{(x+1)(x-1)\left(\sqrt[3]{x^2}-\sqrt[3]{x}+1\right)}$$

$$= \frac{\sqrt{x^2+3}+2}{(x-1)\left(\sqrt[3]{x^2}-\sqrt[3]{x}+1\right)} \qquad (x\neq -1)$$

$$\lim_{x\to -1} \frac{\sqrt[3]{x}+1}{\sqrt{x^2+3}-2} = \lim_{x\to 1} \frac{\sqrt{x^2+3}+2}{(x-1)\left(\sqrt[3]{x^2}-\sqrt[3]{x}+1\right)} = \frac{2+2}{-2\cdot 3} = -\frac{2}{3}$$

$$\boxed{\lim_{x\to -1} \frac{\sqrt[3]{x}+1}{\sqrt{x^2+3}-2} = -\frac{2}{3}}$$

75. គណនា $\lim\limits_{x\to +\infty} \dfrac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+1}}$

$$\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+1}} = \frac{\sqrt{x}\sqrt{\frac{x+\sqrt{x+\sqrt{x}}}{x}}}{\sqrt{x}\sqrt{\frac{x+1}{x}}} = \frac{\sqrt{\frac{x+\sqrt{x+\sqrt{x}}}{x}}}{\sqrt{\frac{x+1}{1}}}$$

$$= \frac{\sqrt{1+\sqrt{\frac{x+\sqrt{x}}{x^2}}}}{\sqrt{1+\frac{1}{x}}} \qquad (x\neq 0)$$

$$\lim_{x\to +\infty} \frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+1}} = \lim_{x\to\infty} \frac{\sqrt{1+\sqrt{\frac{x+\sqrt{x}}{x^2}}}}{\sqrt{1+\frac{1}{x}}} = 1$$

$$\left(\text{ព្រោះ: } \lim_{x\to+\infty}\sqrt{\frac{x+\sqrt{x}}{x^2}}=0\right)$$

$$\boxed{\lim_{x\to+\infty}\frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+1}}=1}$$

76 គណនា $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}}$

$$\frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}}=\frac{\sqrt{x}\left(1+\frac{\sqrt[3]{x}}{\sqrt{x}}+\frac{\sqrt[4]{x}}{\sqrt{x}}\right)}{\sqrt{x}\left(\sqrt{2+\frac{1}{x}}\right)}=\frac{1+\frac{\sqrt[3]{x}}{\sqrt{x}}+\frac{\sqrt[4]{x}}{\sqrt{x}}}{\sqrt{2+\frac{1}{x}}}\quad(x\neq 0)$$

$$\lim_{x\to\infty}\frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}}=\lim_{x\to+\infty}\frac{1+\frac{\sqrt[3]{x}}{\sqrt{x}}+\frac{\sqrt[4]{x}}{\sqrt{x}}}{\sqrt{2+\frac{1}{x}}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$$

$$\left(\text{ព្រោះ: } \lim_{x\to+\infty}\frac{\sqrt[3]{x}}{\sqrt{x}}=\lim_{x\to+\infty}\frac{\sqrt[4]{x}}{\sqrt{x}}=0\right)$$

$$\boxed{\lim_{x\to+\infty}\frac{\sqrt{x}+\sqrt[3]{x}+\sqrt[4]{x}}{\sqrt{2x+1}}=\frac{\sqrt{2}}{2}}$$

77. គណនា $\lim\limits_{x\to+\infty}\dfrac{2x^2-5+\sqrt{x^4-3x+1}}{(x-1)+\sqrt[3]{4x^6+3x-2}}$

$$\frac{2x^2-5+\sqrt{x^4-3x+1}}{(x-1)+\sqrt[3]{4x^6+3x-2}}=\frac{x^2\left(2-\frac{5}{x^2}+\sqrt{1-\frac{3}{x^3}+\frac{1}{x^4}}\right)}{x^2\left(\frac{1}{x}-\frac{1}{x^2}+\sqrt[3]{4+\frac{3}{x^5}-\frac{2}{x^6}}\right)}$$

$$=\frac{2-\frac{5}{x^2}+\sqrt{1-\frac{3}{x^3}+\frac{1}{x^4}}}{\frac{1}{x}-\frac{1}{x^2}+\sqrt[3]{4+\frac{3}{x^5}-\frac{2}{x^6}}}\quad(x\neq 0)$$

$$\lim_{x\to\infty}\frac{2x^2-5+\sqrt{x^4-3x+1}}{(x+1)+\sqrt[3]{4x^6+3x-2}}=\lim_{x\to\infty}\frac{2-\frac{5}{x^2}+\sqrt{1-\frac{3}{x^3}+\frac{1}{x^4}}}{\frac{1}{x}-\frac{1}{x^2}+\sqrt[3]{4+\frac{3}{x^5}-\frac{2}{x^6}}}=\frac{3}{\sqrt[3]{4}}$$

$$\boxed{\lim_{x\to\infty}\frac{2x^2-5+\sqrt{x^4-3x+1}}{(x-1)+\sqrt[3]{4x^6+3x-2}}=\frac{3}{\sqrt[3]{4}}}$$

78. គណនា $\lim\limits_{x\to 1}\left(\dfrac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}}$

$$\frac{1-\sqrt{x}}{1-x}=\frac{1-\sqrt{x}}{(1+\sqrt{x})(1-\sqrt{x})}=\frac{1}{1+\sqrt{x}}\quad(x\neq 1)$$

$$\lim_{x\to 1}\frac{1-\sqrt{x}}{1-x}=\lim_{x\to 1}\frac{1}{1+\sqrt{x}}=\frac{1}{2}$$

$$\lim_{x\to 1}\frac{1+x}{2+x}=\frac{2}{3}$$

$$\lim_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\frac{1+\sqrt{x}}{1+x}}=\left(\frac{2}{3}\right)^{\frac{1}{2}}=\frac{\sqrt{2}}{\sqrt{3}}=\frac{\sqrt{6}}{3}$$

$$\boxed{\lim_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}}=\frac{\sqrt{6}}{3}}$$

79. គណនា $\displaystyle\lim_{x\to\infty}\left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{3}}{1-x}}$

ដោយ $\displaystyle\lim_{x\to\infty}\frac{1+x}{2+x}=\lim_{x\to\infty}\frac{x\left(\frac{1}{x}+1\right)}{x\left(\frac{2}{x}+1\right)}=1 \qquad (x\neq 0)$

ហើយ $\displaystyle\lim_{x\to\infty}\frac{1-\sqrt{x}}{1-x}=\lim_{x\to\infty}\frac{1}{1+\sqrt{x}}=0$

$$\boxed{\lim_{x\to\infty}\left(\frac{1+x}{2+x}\right)^{\frac{1-\sqrt{x}}{1-x}}=1^0=1}$$

80. គណនា $\displaystyle\lim_{x\to 0}\left(\frac{x^3-3x+1}{x-4}+1\right)$

$$\lim_{x\to 0}\left(\frac{x^3-3x+1}{x-4}+1\right)=-\frac{1}{4}+1=\frac{3}{4}$$

ដូចនេះ $\displaystyle\boxed{\lim_{x\to 0}\left(\frac{x^3-3x+1}{x-4}+1\right)=\frac{3}{4}}$

81. គណនា $\displaystyle\lim_{x\to 1/2}\frac{8x^3-1}{6x^2-5x+1}$

$$\frac{8x^3-1}{6x^2-5x+1}=\frac{(2x-1)(4x^2+2x+1)}{(3x-1)(2x-1)}=\frac{4x^2+2x+1}{3x-1}\qquad \left(x\neq\frac{1}{2}\right)$$

$$\lim_{x\to\frac{1}{2}}\frac{8x^3-1}{6x^2-5x+1}=\lim_{x\to 1/2}\frac{4x^2+2x+1}{3x-1}=\frac{1+1+1}{\frac{3}{2}-1}=\frac{3}{\frac{1}{2}}=6$$

$$\boxed{\lim_{x\to\frac{1}{2}}\frac{8x^3-1}{6x^2-5x+1}=6}$$

82. គណនា $\displaystyle\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)$

$$\frac{1}{1-x}-\frac{3}{1-x^3}=\frac{1-x+x^2-3}{(1-x)(1-x+x^2)}=\frac{x^2-x-2}{(1-x)(1-x+x^2)}$$

$$=\frac{(x-1)(x+2)}{-(x-1)(1+x+x^2)}=-\frac{(x+2)}{(1+x+x^2)}\qquad (x\neq 1)$$

$$\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)=\lim_{x\to 1}\frac{-(x+2)}{(1+x+x^2)}=-1$$

$$\boxed{\lim_{x\to 1}\left(\frac{1}{1-x}-\frac{3}{1-x^3}\right)=-1}$$

83. គណនា $\displaystyle\lim_{x\to\infty}\frac{x^4-5x}{x^2-3x+1}$

$$\frac{x^4-5x}{x^2-3x+1}=\frac{x^2\left(x^2-\frac{5}{x}\right)}{x^2\left(1-\frac{3}{x}+\frac{1}{x^2}\right)}=\frac{x^2-\frac{5}{x}}{1-\frac{3}{x}+\frac{1}{x^2}}\qquad (x\neq 0)$$

$$\lim_{x\to\infty}\frac{x^4-5x}{x^2-3x+1}=\lim_{x\to\infty}\frac{x^2-\frac{5}{x}}{1-\frac{3}{x}+\frac{1}{x^2}}=+\infty$$

$$\boxed{\lim_{x\to\infty}\frac{x^4-5x}{x^2-3x+1}=+\infty}$$

84. គណនា $\lim\limits_{x\to\infty}\dfrac{1+x-3x^3}{1+x^2+3x^3}$

$$\frac{1+x-3x^3}{1+x^2+3x^3}=\frac{x^3\left(\frac{1}{x^3}+\frac{1}{x^2}-3\right)}{x^3\left(\frac{1}{x^3}+\frac{1}{x}+3\right)}=\frac{\frac{1}{x^3}+\frac{1}{x^2}-3}{\frac{1}{x^3}+\frac{1}{x}+3}\quad(x\neq 0)$$

$$\lim_{x\to\infty}\frac{1+x-3x^3}{1+x^2+3x^3}=\lim_{x\to\infty}\frac{\frac{1}{x^3}+\frac{1}{x^2}-3}{\frac{1}{x^3}+\frac{1}{x}+3}=-1$$

$$\boxed{\lim_{x\to\infty}\frac{1-x-3x^3}{1+x^2+x^3}=-1}$$

85. គណនា $\lim\limits_{x\to\infty}\left(\dfrac{x^3}{2x^2-1}-\dfrac{x^2}{2x+1}\right)$

$$\frac{x^3}{2x^2-1}-\frac{x^2}{2x+1}=\frac{2x^4+x^3-2x^4+x^2}{4x^3+2x^2-2x-1}=\frac{x^3+x^2}{4x^3+2x^2-2x-1}$$

$$=\frac{x^3\left(1+\frac{1}{x}\right)}{x^3\left(4+\frac{2}{x}-\frac{2}{x^2}-\frac{1}{x^3}\right)}=\frac{1+\frac{1}{x}}{4+\frac{2}{x}-\frac{2}{x^2}-\frac{1}{x^3}}$$

$(x\neq 0)$

$$\boxed{\lim_{x\to\infty}\left(\frac{x^3}{2x^2-1}-\frac{x^2}{2x+1}\right)=\frac{1}{4}}$$

86. គណនា $\lim\limits_{x\to\infty}\dfrac{\sqrt{x^2+1}-\sqrt[3]{x^2+1}}{\sqrt[4]{x^4+1}-\sqrt[5]{x^4+1}}$

$$\frac{\sqrt{x^2+1}-\sqrt[3]{x^2+1}}{\sqrt[4]{x^4+1}-\sqrt[5]{x^4+1}}=\frac{x\left(\sqrt{1+\frac{1}{x^2}}-\sqrt[3]{\frac{1}{x}+\frac{1}{x^3}}\right)}{x\left(\sqrt[4]{1+\frac{1}{x^4}}-\sqrt[5]{\frac{1}{x}+\frac{1}{x^5}}\right)}$$

$$=\frac{\sqrt{1+\frac{1}{x^2}}-\sqrt[3]{\frac{1}{x}-\frac{1}{x^3}}}{\sqrt[4]{1+\frac{1}{x^4}}-\sqrt[5]{\frac{1}{x}-\frac{1}{x^5}}}\ ;\quad(x\neq 0)$$

$$\lim_{x\to\infty}\frac{\sqrt{x^2+1}-\sqrt[3]{x^2+1}}{\sqrt[4]{x^4+1}-\sqrt[5]{x^4+1}}=\lim_{x\to\infty}\frac{\sqrt{1+\frac{1}{x^2}}-\sqrt[3]{\frac{1}{x}-\frac{1}{x^3}}}{\sqrt[4]{1+\frac{1}{x^4}}-\sqrt[5]{\frac{1}{x}-\frac{1}{x^5}}}=1$$

$$\boxed{\lim_{x\to\infty}\frac{\sqrt{x^2+1}-\sqrt[3]{x^2+1}}{\sqrt[4]{x^4+1}-\sqrt[5]{x^4+1}}=1}$$

87. គណនា $\lim\limits_{x\to\infty}\dfrac{\sqrt[3]{x^4+3}-\sqrt[5]{x^3+4}}{\sqrt[3]{x^7+1}}$

$$\frac{\sqrt[3]{x^4+3}-\sqrt[5]{x^3+4}}{\sqrt[3]{x^7+1}}=\frac{\sqrt[3]{x^4}\left(\sqrt[3]{1+\frac{3}{x^4}}-\frac{1}{\sqrt[15]{x^{11}}}\cdot\sqrt[5]{1+\frac{4}{x^5}}\right)}{\sqrt[3]{x^4}\left(\sqrt[3]{x^3+\frac{1}{x^4}}\right)}$$

$$= \frac{\sqrt[3]{1+\frac{3}{x^4}} - \frac{1}{\sqrt[5]{x^{11}}}\cdot\sqrt[5]{1+\frac{4}{x^5}}}{\sqrt[3]{x^3+\frac{1}{x^4}}}$$

ដោយ $x \to \infty \Rightarrow \dfrac{1}{\sqrt[5]{x^{11}}} \to 0$

$$\lim_{x\to\infty} \frac{\sqrt[3]{x^4+3}-\sqrt[5]{x^3+4}}{\sqrt[3]{x^7+1}} = \lim_{x\to\infty}\frac{\sqrt[3]{1+\frac{3}{x^4}}}{\sqrt[3]{x^3+\frac{1}{x^4}}} = \frac{1}{\infty} = 0$$

$$\boxed{\lim_{x\to\infty} \frac{\sqrt[3]{x^4+3}-\sqrt[5]{x^3+4}}{\sqrt[3]{x^7+1}} = 0}$$

88. គណនា $\displaystyle\lim_{x\to 0} \frac{\sin x \cdot \sin 2x \ldots \sin nx}{x^n}$

$$\frac{\sin x \cdot \sin 2x \ldots \sin nx}{x^n} = \frac{\sin x}{x}\cdot\frac{\sin 2x}{x}\cdots\cdots\frac{\sin nx}{x}$$

ដោយ $\displaystyle\lim_{x\to 0}\frac{\sin x}{x} = 1$

$$\lim_{x\to 0}\frac{\sin x \cdot \sin 2x \ldots \sin nx}{x^n} = \lim_{x\to 0}\left(\frac{\sin x}{x}\cdot\frac{\sin 2x}{x}\cdots\frac{\sin nx}{x}\right)$$

$$= 1.2.3\ldots n = n!$$

$$\boxed{\lim_{x\to 0}\frac{\sin x \cdot \sin 2x \ldots \sin nx}{x^n} = n!}$$

89. គណនា $\displaystyle\lim_{x\to 0}\frac{\cos x - \cos 3x}{\sin^2 x}$

$$\frac{\cos x - \cos 3x}{\sin^2 x} = \frac{-2\sin 2x \sin(-x)}{\sin^2 x} = \frac{4\sin^2 x\cos x}{\sin^2 x} = 4\cos x$$

$$\lim_{x\to 0}\frac{\cos x - \cos x}{\sin^2 x} = \lim_{x\to 0} 4\cos x = 4$$

$$\boxed{\lim_{x\to 0}\frac{\cos x - \cos 3x}{\sin^2 x} = 4}$$

90 គណនា $\displaystyle\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin 2x}$

$$\frac{1-\cos^3 x}{x\sin 2x} = \frac{(1-\cos x)(1+\cos x+\cos^2 x)}{2x\sin x\cos x}$$

$$= \frac{2\sin^2\frac{x}{2}(1+\cos x+\cos^2 x)}{4x\cos\frac{x}{2}\cos x\sin\frac{x}{2}} = \frac{2\sin\frac{x}{2}}{\frac{x}{2}\times 2}\cdot\frac{1+\cos x+\cos^2 x}{4\cos\frac{x}{2}\cos x}$$

$$\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin 2x} = \lim_{x\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\times\cdot\lim_{x\to 0}\frac{1+\cos x+\cos^2 x}{4\cos\frac{x}{2}\cos x} = \frac{3}{4}$$

$$\boxed{\lim_{x\to 0}\frac{1-\cos^3 x}{x\sin 2x} = \frac{3}{4}}$$

91. គណនា $\displaystyle\lim_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}$

$$\frac{1+\sin x-\cos x}{1-\sin x-\cos x}=\frac{2\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{2\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$=\frac{2\sin\frac{x}{2}\left(\sin\frac{x}{2}+\cos\frac{x}{2}\right)}{2\sin\frac{x}{2}\left(\sin\frac{x}{2}-\cos\frac{x}{2}\right)}=\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\sin\frac{x}{2}-\cos\frac{x}{2}}$$

$$\lim_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}=\lim_{x\to 0}\frac{\sin\frac{x}{2}+\cos\frac{x}{2}}{\sin\frac{x}{2}-\cos\frac{x}{2}}=-1$$

$$\boxed{\lim_{x\to 0}\frac{1+\sin x-\cos x}{1-\sin x-\cos x}=-1}$$

92. គណនា $\lim_{x\to\pi/2}\left(\frac{1}{\cos x}-\operatorname{tg}x\right)$

$$\frac{1}{\cos x}-\operatorname{tg}x=\frac{1-\sin x}{\cos x}=\frac{1-\cos\left(\pi/2-x\right)}{\sin\left(\pi/2-x\right)}$$

$$=\frac{2\sin^2\left(\pi/4-x/2\right)}{2\sin\left(\pi/4-x/2\right)\cos\left(\pi/4-x/2\right)}$$

$$=\frac{\sin\left(\frac{\pi}{4}-\frac{x}{2}\right)}{\cos\left(\frac{\pi}{4}-\frac{x}{2}\right)}=\operatorname{tg}\left(\frac{\pi}{4}-\frac{x}{2}\right)$$

$$\lim_{x\to\pi/2}\left(\frac{1}{\cos x}-\operatorname{tg}x\right)=\lim_{x\to\pi/2}\operatorname{tg}\left(\frac{\pi}{4}-\frac{x}{2}\right)=0$$

$$\boxed{\lim_{x\to\pi/2}\left(\frac{1}{\cos x}-\operatorname{tg}x\right)=0}$$

93. គណនា $\lim_{x\to\pi/2}\left(\frac{\pi}{2}-x\right)\operatorname{tg}x$

$$\left(\frac{\pi}{2}-x\right)\operatorname{tg}x=\left(\frac{\pi}{2}-x\right)\frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}$$

$$\lim_{x\to\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)\operatorname{tg}x=\lim_{x\to\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)\frac{\cos\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)}=1$$

$$\boxed{\lim_{x\to\frac{\pi}{2}}\left(\frac{\pi}{2}-x\right)\operatorname{tg}x=1}$$

94. $\lim_{\alpha\to 0}\frac{(1-\cos\alpha)^2}{\operatorname{tg}^3\alpha-\sin^3\alpha}$

ចម្លើយ

$$\frac{(1-\cos\alpha)^2}{\operatorname{tg}^3\alpha-\sin^3\alpha}=\frac{(1-\cos\alpha)^2}{\operatorname{tg}^3\alpha(1-\cos^3\alpha)}=\frac{\cos^3\alpha(1-\cos\alpha)^2}{\sin^3\alpha(1-\cos\alpha)(1+\cos\alpha+\cos^2\alpha)}$$

$$=\frac{\cos^3\alpha(1-\cos\alpha)}{\sin^3\alpha(1+\cos\alpha+\cos^2\alpha)}=\frac{\cos^3\alpha\cdot 2\sin^2\alpha/2}{(2\sin\alpha/2\cdot\cos\alpha/2)^3(1+\cos\alpha+\cos^2\alpha)}$$

$$=\frac{2\sin^2\alpha/2\cdot\cos^3\alpha}{8\sin^3\alpha/2\cdot\cos^3\alpha/2\,(1+\cos\alpha+\cos^2\alpha)}$$

$$=\frac{\cos^3\alpha}{4\sin\alpha/2\cdot\cos^3\frac{\alpha}{2}\,(1+\cos\alpha+\cos^2\alpha)}$$

$$\lim_{\alpha\to 0}\frac{(1-\cos\alpha)^2}{\operatorname{tg}^3\alpha-\sin^3\alpha}=\lim_{\alpha\to 0}\frac{\cos^3\alpha}{4\sin\frac{\alpha}{2}\cos^3\frac{\alpha}{2}(1+\cos\alpha+\cos^2\alpha)}=\infty$$

$$\boxed{\lim_{\alpha\to 0}\frac{(1-\cos\alpha)^2}{tg^3\alpha-\sin^3\alpha}=\infty}$$

95. គណនា $\lim\limits_{\alpha\to\pi}\dfrac{\sin\alpha}{1-\frac{\alpha^2}{\pi^2}}$

$$\frac{\sin\alpha}{1-\frac{\alpha^2}{\pi^2}}=\frac{\sin\alpha}{\frac{\pi^2-\alpha^2}{\pi^2}}=\frac{\pi^2\sin\alpha}{(\pi+\alpha)(\pi-\alpha)}=\frac{\pi^2\sin(\pi-\alpha)}{(\pi+\alpha)(\pi-\alpha)}$$

$$=\frac{\pi^2}{\pi+\alpha}$$ (ព្រោះ: $\sin(\pi-\alpha)\approx(\pi-\alpha)$ កាលណា $\alpha\to\pi$)

$$\lim_{\alpha\to\pi}\frac{\sin\alpha}{1-\frac{\alpha^2}{\pi^2}}=\lim_{\alpha\to\pi}\frac{\pi^2}{\pi+\alpha}=\frac{\pi}{2}$$

$$\boxed{\lim_{\alpha\to\pi}\frac{\sin\alpha}{1-\frac{\alpha^2}{\pi^2}}=\frac{\pi}{2}}$$

96. គណនា $\lim\limits_{x\to\pi}\dfrac{\sin 3x}{\sin 2x}$

គន្លឹះ

$\sin 3x=\sin(2x+x)=\sin 2x\cos x+\sin x\cos 2x$

$=2\sin x\cdot\cos^2 x+\sin x(1-2\sin^2 x)$

$=2\sin x(1-\sin^2 x)+\sin x(1-2\sin^2 x)$

$=3\sin x-4\sin^3 x$

$$\frac{\sin 3x}{\sin 2x}=\frac{3\sin x-4\sin^3 x}{2\sin x\cos x}=\frac{\sin x(3-4\sin^2 x)}{2\sin x\cos x}$$

$$=\frac{3-4\sin^2 x}{2\cos x}$$

$$\lim_{x\to\pi}\frac{\sin 3x}{\sin 2x}=\lim_{x\to\pi}\frac{3-4\sin^2 x}{2\cos x}=\frac{3}{-2}=-\frac{3}{2}$$

$$\boxed{\lim_{x\to\pi}\frac{\sin 3x}{\sin 2x}=-\frac{3}{2}}$$

97. គណនា $\lim\limits_{y\to a}\left(\sin\dfrac{y-a}{2}\,tg\dfrac{\pi y}{2a}\right)$

$$\sin\frac{y-a}{2}\,tg\frac{\pi y}{2a}=\frac{\sin\frac{y-a}{2}\cdot\sin\frac{\pi y}{2a}}{\cos\frac{\pi y}{2a}}=\frac{\sin\frac{y-a}{2}\cdot\sin\frac{\pi y}{2a}}{\sin\left(\frac{\pi}{2}-\frac{\pi y}{2a}\right)}$$

$$\lim_{y\to a}\sin\frac{y-a}{2}\cdot tg\frac{\pi y}{2a}=\lim_{x\to a}\frac{\frac{\sin\frac{y-a}{2}}{\frac{y-a}{2}}\times\frac{y-a}{2}\cdot\sin\frac{\pi y}{2a}}{\frac{\sin\left(\frac{\pi}{2}-\frac{\pi y}{2a}\right)}{\left(\frac{\pi}{2}-\frac{\pi y}{2a}\right)}\cdot\left(\frac{\pi}{2}-\frac{\pi y}{2a}\right)}$$

$$=\frac{y-a}{2}\cdot\frac{2a}{\pi(a-y)}=-\frac{a}{\pi}$$

$$\boxed{\lim_{y\to a}\left(\sin\frac{y-a}{2}\cdot tg\frac{\pi y}{2a}\right)=-\frac{a}{\pi}}$$

លំហាត់ ៩៨

$$\lim_{x \to \frac{\pi}{6}} \frac{\sin\left(x - \frac{\pi}{6}\right)}{\frac{\sqrt{3}}{2} - \cos x}$$

យើងមាន

$$\frac{\sin\left(x - \frac{\pi}{6}\right)}{\frac{\sqrt{3}}{2} - \cos x} = \frac{\sin\left(x - \frac{\pi}{6}\right)}{\cos\frac{\pi}{6} - \cos x} = \frac{2\sin\left(\frac{x}{2} - \frac{\pi}{12}\right)\cos\left(\frac{x}{2} - \frac{\pi}{12}\right)}{-2\sin\left(\frac{\pi}{12} - \frac{x}{2}\right)\sin\left(\frac{\pi}{12} + \frac{x}{2}\right)}$$

$$= \frac{\cos\left(\frac{x}{2} - \frac{\pi}{12}\right)}{\sin\left(\frac{\pi}{12} + \frac{x}{2}\right)} \qquad \left(\text{ព្រោះ: } -\sin\alpha = \sin(-\alpha)\right)$$

$$\Rightarrow \lim_{x \to \frac{\pi}{6}} \frac{\sin\left(x - \frac{\pi}{6}\right)}{\frac{\sqrt{3}}{2} - \cos x} = \lim_{x \to \frac{\pi}{6}} \frac{\cos\left(\frac{x}{2} - \frac{\pi}{12}\right)}{\sin\left(\frac{\pi}{12} + \frac{x}{2}\right)} = \frac{1}{\frac{1}{2}} = 2$$

ដូចនេះ:

$$\boxed{\lim_{x \to \frac{\pi}{6}} \frac{\sin\left(x - \frac{\pi}{6}\right)}{\frac{\sqrt{3}}{2} - \cos x} = 2}$$

លំហាត់ ៩៩

$$\lim_{x \to \frac{\pi}{2}} \left(2x\,\mathrm{tg}\,x - \frac{\pi}{\cos x}\right)$$

យើងមាន

$$2x\,\mathrm{tg}\,x - \frac{\pi}{\cos x} = \frac{2x\sin x - \pi}{\cos x}$$

$$\Rightarrow \lim_{x \to \frac{\pi}{2}} \left(2x\,\mathrm{tg}\,x - \frac{\pi}{\cos x}\right) = \lim_{x \to \frac{\pi}{2}} \frac{(2x\sin x - \pi)'}{(\cos x)'}$$

$$= \lim_{x \to \frac{\pi}{2}} \frac{2\sin x - 2x\cos x}{-\sin x}$$

$$= \frac{2}{-1} = -2$$

ដូចនេះ:

$$\boxed{\lim_{x \to \frac{\pi}{2}} \left(2x\,\mathrm{tg}\,x - \frac{\pi}{\cos x}\right) = -2}$$

100. គណនា $\lim_{x \to 0} \frac{\cos\alpha x - \cos\beta x}{x^2}$

$$\frac{\cos\alpha x - \cos\beta x}{x^2} = \frac{-2\sin\frac{(\alpha+\beta)x}{2} \cdot \sin\frac{(\alpha.\beta)x}{2}}{x^2}$$

$$\frac{-2\sin\frac{(\alpha+\beta)x}{2}}{\frac{(\alpha+\beta)x}{2}} \cdot \frac{\alpha+\beta}{2} \cdot \frac{\sin\frac{(\alpha-\beta)x}{2}}{\frac{(\alpha-\beta)x}{2}} \cdot \frac{\alpha-\beta}{2}$$

$$\lim_{x\to 0}\frac{\cos\alpha x-\cos\beta x}{x^2}=\lim_{x\to 0}\frac{-2\sin\frac{(\alpha+\beta)x}{2}}{\frac{(\alpha+\beta)x}{2}}\cdot\frac{\alpha+\beta}{2}\cdot\frac{\sin\frac{(\alpha-\beta)x}{2}}{\frac{(\alpha-\beta)x}{2}}\cdot\frac{\alpha-\beta}{2}$$

$$=-2\left(\frac{\alpha+\beta}{2}\right)\left(\frac{\alpha-\beta}{2}\right)=\frac{\beta^2-\alpha^2}{2}$$

$$\boxed{\lim_{x\to 0}\frac{\cos\alpha x-\cos\beta x}{x^2}=\frac{\beta^2-\alpha^2}{2}}$$

101. គណនា $\lim_{\alpha\to\beta}\frac{\sin^2\alpha-\sin^2\beta}{\alpha^2-\beta^2}$

$$\frac{\sin^2\alpha-\sin^2\beta}{\alpha^2-\beta^2}=\frac{(\sin\alpha+\sin\beta)(\sin\alpha-\sin\beta)}{\alpha^2-\beta^2}$$

$$=\frac{2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}\cdot 2\cos\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}}{(\alpha+\beta)(\alpha-\beta)}$$

$$\lim_{\alpha\to\beta}\frac{\sin^2\alpha-\sin^2\beta}{\alpha^2-\beta^2}=\lim_{\alpha\to\beta}\frac{2\sin\frac{\alpha+\beta}{2}\cdot\cos\frac{\alpha-\beta}{2}\cdot 2\cos\frac{\alpha+\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}}{(\alpha+\beta)(\alpha-\beta)}$$

$$=\lim_{\alpha\to\beta}\frac{2\sin(\alpha+\beta)\cdot\cos\frac{\alpha-\beta}{2}\cdot\sin\frac{\alpha-\beta}{2}}{(\alpha+\beta)(\alpha-\beta)}$$

$$\lim_{\alpha\to\beta}\frac{\sin(\alpha+\beta)\cos\frac{\alpha-\beta}{2}}{\alpha+\beta}\cdot\frac{\sin\frac{\alpha-\beta}{2}}{\frac{\alpha-\beta}{2}}=\frac{\sin 2\beta}{2\beta}\cdot 1=\frac{\sin 2\beta}{2\beta}$$

$$\boxed{\lim_{\alpha\to\beta}\frac{\sin^2\alpha-\sin^2\beta}{\alpha^2-\beta^2}=\frac{\sin 2\beta}{2\beta}}$$

102. គណនា $\lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\operatorname{tg}x}$

$$\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\operatorname{tg}x}=\frac{(1+\sin x-1+\sin x)\cos x}{\sin x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$

$$=\frac{2\sin x\cos x}{\sin x\left(\sqrt{1+\sin x}+\sqrt{1-\sin x}\right)}$$

$$=\frac{2\cos x}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}$$

$$\lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\operatorname{tg}x}=\lim_{x\to 0}\frac{2\cos x}{\sqrt{1+\sin x}+\sqrt{1-\sin x}}=\frac{2}{2}=1$$

$$\boxed{\lim_{x\to 0}\frac{\sqrt{1+\sin x}-\sqrt{1-\sin x}}{\operatorname{tg}x}=1}$$

103. គណនា $\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2}$

$$\frac{1-\cos x\sqrt{\cos 2x}}{x^2}=\frac{1-\cos^2 x\cos 2x}{x^2\left(1+\cos x\sqrt{\cos 2x}\right)}$$

$$=\frac{1-\cos^2 x(1-2\sin^2 x)}{x^2\left(1+\cos x\sqrt{\cos 2x}\right)}=\frac{1-\cos^2 x+2\sin^2 x\cos^2 x}{x^2\left(1+\cos x\sqrt{\cos 2x}\right)}$$

$$= \frac{\sin^2 x + 2\sin^2 x\cos^2 x}{x^2\left(1+\cos x\cdot\sqrt{\cos 2x}\right)} = \frac{\sin^2 x\,(1+2\cos^2 x)}{x^2\left(1+\cos x\sqrt{\cos 2x}\right)}$$

$$\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2} = \lim_{x\to 0}\frac{\sin^2 x\,(1+2\cos^2 x)}{x^2\left(1+\cos x\sqrt{\cos 2x}\right)} = \frac{3}{2}$$

$$\boxed{\lim_{x\to 0}\frac{1-\cos x\sqrt{\cos 2x}}{x^2} = \frac{3}{2}}$$

104. គណនា $\lim\limits_{x\to 0}\dfrac{1-\cos\frac{x}{4}}{x^2}$

$$\frac{1-\cos\frac{x}{4}}{x^2} = \frac{2\sin^2\frac{x}{8}}{\frac{x^2}{64}\cdot 64} = \frac{1}{32}\left(\frac{\sin\frac{x}{8}}{\frac{x}{8}}\right)^2$$

$$\lim_{x\to 0}\frac{1-\cos x/4}{x^2} = \frac{1}{32}\lim_{x\to 0}\left(\frac{\sin\frac{x}{8}}{\frac{x}{8}}\right)^2 = \frac{1}{32}$$

$$\boxed{\lim_{x\to 0}\frac{1-\cos\frac{x}{4}}{x^2} = \frac{1}{32}}$$

105. គណនា $\lim\limits_{x\to 0}\dfrac{\operatorname{tg} x}{\sin 3x}$

$$\frac{\operatorname{tg} x}{\sin 3x} = \frac{\sin x}{\cos x\,\sin 3x} = \frac{\sin x}{x}\cdot x\cdot\frac{1}{\cos x}\cdot\frac{1}{\frac{\sin 3x}{3x}\cdot 3x}$$

$$= \frac{x}{3x}\cdot\frac{\sin x}{x}\cdot\frac{1}{\cos x}\cdot\frac{1}{\frac{\sin 3x}{3x}} = \frac{1}{3}\cdot\frac{1}{\cos x}\cdot\frac{\sin x}{x}\cdot\frac{1}{\frac{\sin 3x}{3x}}$$

$$\lim_{x\to 0}\frac{\operatorname{tg} x}{\sin 3x} = \lim_{x\to 0}\frac{1}{3}\cdot\frac{1}{\cos x}\cdot\frac{\sin x}{x}\cdot\frac{1}{\frac{\sin 3x}{3x}} = \frac{1}{3}$$

$$\boxed{\lim_{x\to 0}\frac{\operatorname{tg} x}{\sin 3x} = \frac{1}{3}}$$

106. គណនា $\lim\limits_{x\to \pi}\dfrac{\operatorname{tg} x}{\sin 2x}$

របៀបទី១ : តាង $X = x - \pi \Rightarrow x = X + \pi$

$x \to \pi \Rightarrow X \to 0$

$$\frac{\operatorname{tg} x}{\sin 2x} = \frac{\operatorname{tg}(X+\pi)}{\sin 2(X+\pi)} = \frac{\operatorname{tg} X}{\sin(2X+2\pi)} = \frac{\operatorname{tg} X}{\sin 2X} = \frac{\sin X}{\cos X\,\sin 2X}$$

$$= \frac{\sin X}{X}\cdot X\cdot\frac{1}{\cos X}\cdot\frac{1}{\frac{\sin 2X}{2X}\cdot 2X} = \frac{X}{2X}\cdot\frac{\sin X}{X}\cdot\frac{1}{\cos X}\cdot\frac{1}{\frac{\sin 2X}{2X}}$$

$$= \frac{1}{2}\cdot\frac{\sin X}{X}\cdot\frac{1}{\cos X}\cdot\frac{1}{\frac{\sin 2X}{2X}} \qquad X\to 0,\; X\neq 0$$

$$\lim_{x\to \pi}\frac{\operatorname{tg} x}{\sin 2x} = \lim_{X\to 0}\frac{1}{2}\cdot\frac{\sin X}{X}\cdot\frac{1}{\cos X}\cdot\frac{1}{\frac{\sin 2X}{2X}} = \frac{1}{2}$$

$$\boxed{\lim_{x\to \pi}\frac{\operatorname{tg} x}{\sin 2x} = \frac{1}{2}}$$

របៀបទី២

$$\frac{\operatorname{tg} x}{\sin 2x} = \frac{\sin x}{\cos x\cdot 2\sin x\cos x} = \frac{1}{2\cos^2 x}$$

$$\lim_{x\to 0}\frac{tg\,x}{\sin 2x}=\lim_{x\to 0}\frac{1}{2\cos^2 x}=\frac{1}{2}$$

$$\boxed{\lim_{x\to 0}\frac{tg\,x}{\sin 2x}=\frac{1}{2}}$$

107. គណនា $\displaystyle\lim_{x\to \pi/4}\frac{\cos 2x}{1-\sqrt{2}\sin x}$

$$\frac{\cos 2x}{1-\sqrt2\sin x}=\frac{1-2\sin^2 x}{1-\sqrt2\sin x}=\frac{(1-\sqrt2\sin x)(1+\sqrt2\sin x)}{1-\sqrt2\sin x}$$

$$=1+\sqrt2\sin x$$

$$\lim_{x\to \pi/4}\frac{\cos 2x}{1-\sqrt2\sin x}=\lim_{x\to\pi/4}(1+\sqrt2\sin x)=1+\sqrt2\cdot\frac{\sqrt2}{2}=2$$

$$\boxed{\lim_{x\to \pi/4}\frac{\cos 2x}{1-\sqrt2\sin x}=2}$$

108. គណនា $\displaystyle\lim_{x\to \pi/4}\frac{\sin 7x+\cos 7x}{\sin 9x-\cos 9x}$

តាង $x=\frac{\pi}{4}+t \Rightarrow t=x-\frac{\pi}{4}$, $x\to\frac{\pi}{4}\Rightarrow t\to 0$

$$\frac{\sin 7x+\cos 7x}{\sin 9x-\cos 9x}=\frac{\sin\left(\frac{7\pi}{4}-7t\right)+\cos\left(\frac{7\pi}{4}+7t\right)}{\sin\left(\frac{9\pi}{4}+9t\right)-\cos\left(\frac{9\pi}{4}+9t\right)}$$

$$=\frac{\sin\left[\left(7t-\frac{\pi}{4}\right)+2\pi\right]+\cos\left[\left(7t-\frac{\pi}{4}\right)+2\pi\right]}{\sin\left[\left(9t+\frac{\pi}{4}\right)+2\pi\right]-\cos\left[\left(9t+\frac{\pi}{4}\right)+2\pi\right]}$$

$$=\frac{\sin\left(7t-\frac{\pi}{4}\right)+\cos\left(7t-\frac{\pi}{4}\right)}{\sin\left(9t+\frac{\pi}{4}\right)-\cos\left(9t+\frac{\pi}{4}\right)}$$

$$=\frac{\sin\left(7t-\frac{\pi}{4}\right)+\sin\left(7t+\frac{\pi}{4}\right)}{\sin\left(9t+\frac{\pi}{4}\right)-\sin\left(\frac{\pi}{4}-9t\right)}$$

$$\lim_{x\to\pi/4}\frac{\sin 7x+\cos 7x}{\sin 9x-\cos 9x}=\lim_{t\to 0}\frac{\sin\left(7t-\frac{\pi}{4}\right)+\sin\left(7t+\frac{\pi}{4}\right)}{\sin\left(9t+\frac{\pi}{4}\right)-\sin\left(\frac{\pi}{4}-9t\right)}$$

$$=\lim_{t\to 0}\left(\frac{\sin 7t}{7t}\cdot\frac{9t}{\sin 9t}\cdot\frac{7}{9}\right)=\frac{7}{9}$$

$$\boxed{\lim_{x\to \pi/4}\frac{\sin 7x+\cos 7x}{\sin 9x-\cos 9x}=\frac{7}{9}}$$

109. គណនា $\displaystyle\lim_{n\to+\infty}\left(2^n tg\frac{\pi}{2^n}\right)$

តាង $t=\frac{\pi}{2^n}\Rightarrow 2^n=\frac{\pi}{t}$; $n\to\infty$ នោះ $t\to 0$

$$\lim_{n\to+\infty}\left(2^n tg\frac{\pi}{2^n}\right)=\lim_{t\to 0}\left(\frac{\pi}{t}\cdot tg\,t\right)=\lim_{t\to 0}\left(\frac{tg\,t}{t}\cdot\pi\right)=\pi$$

$$\boxed{\lim_{n\to+\infty}\left(2^n tg\frac{\pi}{2^n}\right)=\pi}$$

110. គណនា $\lim\limits_{n\to+\infty}\left(\frac{1}{2^n}\cot g\frac{x}{2^n}-2\cot g\,2x\right)$

$$\lim_{n\to+\infty}\left(\frac{1}{2^n}\cot g\frac{x}{2^n}-2\cot g\,2x\right)=\lim_{n\to+\infty}\frac{1}{2^n}\cdot\cot g\frac{x}{2^n}-\lim_{n\to+\infty}2\cot g\,2x$$

$$=\lim_{n\to+\infty}\frac{1}{2^n}\cdot\frac{1}{tg\frac{x}{2^n}}-2\cot g\,2x$$

គណនា $\lim\limits_{n\to+\infty}\left(\frac{1}{2^n}\cdot\frac{1}{tg\frac{x}{2^n}}\right)$

តាង $t=\frac{x}{2^n}\Rightarrow 2^n=\frac{x}{t}$, $n\to+\infty$ នោះ $t\to 0$

$$\lim_{n\to+\infty}\left(\frac{1}{2^n}\cdot\frac{1}{tg\frac{x}{2^n}}\right)=\lim_{t\to 0}\frac{t}{x\,tg\,t}=\frac{1}{x}$$

$$\boxed{\lim_{n\to+\infty}\left(\frac{1}{2^n}\cdot\cot g\frac{x}{2^n}-2\cot g\,2x\right)=\frac{1}{x}-2\cot g\,2x}$$

111. គណនា $\lim\limits_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{\cos 2x}$

$$\frac{\cos x-\sin x}{\cos 2x}=\frac{\cos x-\sin x}{\cos^2 x-\sin^2 x}=\frac{\cos x-\sin x}{(\cos x+\sin x)(\cos x-\sin x)}$$

$$=\frac{1}{\cos x+\sin x}\quad\left(x\to\frac{\pi}{4},\ x\neq\frac{\pi}{4}\right)$$

$$\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{\cos 2x}=\lim_{x\to\frac{\pi}{4}}\frac{1}{\cos x+\sin x}=\frac{1}{\frac{2\sqrt{2}}{2}}=\frac{\sqrt{2}}{2}$$

$$\boxed{\lim_{x\to\frac{\pi}{4}}\frac{\cos x-\sin x}{\cos 2x}=\frac{\sqrt{2}}{2}}$$

112. គណនា $\lim\limits_{x\to 0}\frac{\cos ax-\cos bx\cdot\cos ex}{x^2}$

$$\frac{\cos ax-\cos bx\cdot\cos ex}{x^2}=\frac{\cos ax-\cos bx+\cos bx(1-\cos ex)}{x^2}$$

$$=\frac{-2\sin\frac{(a-b)x}{2}\cdot\sin\frac{(a+b)x}{2}+2\cos bx\cdot\sin^2\frac{ex}{2}}{x^2}$$

$$\lim_{x\to 0}\frac{\cos ax-\cos bx\cdot\cos ex}{x^2}$$

$$=2\lim_{x\to 0}\frac{\cos bx\cdot\sin^2\frac{ex}{2}}{x^2}-2\lim_{x\to 0}\frac{\sin\frac{(a-b)x}{2}\cdot\sin\frac{(a+b)x}{2}}{x^2}$$

$$=2\lim_{x\to 0}\frac{e^2}{4}\cos bx\left(\frac{\sin\frac{ex}{2}}{\frac{ex}{2}}\right)^2-2\lim_{x\to 0}\frac{a^2-b^2}{4}\cdot\frac{\sin\frac{(a-b)x}{2}}{\frac{(a-b)x}{2}}\cdot\frac{\sin\frac{(a+b)x}{2}}{\frac{(a+b)x}{2}}$$

$$=2\cdot\frac{e^2}{4}\cdot 1\cdot 1-2\cdot\frac{a^2-b^2}{4}\cdot 1\cdot 1$$

$$=\frac{e^2}{2}-\frac{a^2-b^2}{2}=\frac{e^2+b^2-a^2}{2}$$

$$\boxed{\lim_{x\to 0}\frac{\cos ax-\cos bx\cdot\cos ex}{x^2}=\frac{e^2+b^2-a^2}{2}}$$

113. គណនា $\lim\limits_{x\to 0}\dfrac{\operatorname{tg}x-\sin x}{x^3}$

$$\frac{\operatorname{tg}x-\sin x}{x^3}=\frac{\frac{\sin x}{\cos x}-\sin x}{x^3}=\frac{\sin x(1-\cos x)}{x^3.\cos x}$$

$x\to 0$ នោះ: $\sin x\simeq x$, $1-\cos x\simeq\frac{1}{2}x^2$

$$\lim_{x\to 0}\frac{\operatorname{tg}x-\sin x}{x^3}=\lim_{x\to 0}\frac{\sin x(1-\cos x)}{x^3\cos x}=\lim_{x\to 0}\frac{x.\frac{1}{2}.x^2}{x^3.\cos x}$$

$$=\lim_{x\to 0}\frac{1}{2\cos x}=\frac{1}{2}$$

$$\boxed{\lim_{x\to 0}\frac{\operatorname{tg}x-\sin x}{x^3}=\frac{1}{2}}$$

114. គណនា $\lim\limits_{x\to\pi/3}\dfrac{\sin 3x}{1-2\cos x}$

តាង $U=x-\frac{\pi}{3}\Rightarrow x=U+\frac{\pi}{3}$

$x\to\pi/3\Rightarrow U\to 0$

$$\frac{\sin 3x}{1-2\cos x}=\frac{\sin 3(U+\pi/3)}{1-2\cos(U+\pi/3)}=\frac{\sin(3U+\pi)}{1-2\cos(U+\pi/3)}$$

$$=\frac{\sin(3U+\pi)}{1-2\left[\cos U.\cos\frac{\pi}{3}-\sin U.\sin\frac{\pi}{3}\right]}$$

$$=\frac{-\sin 3U}{1-2\left[\frac{1}{2}\cos U-\frac{\sqrt{3}}{2}\sin U\right]}$$

$$=-\frac{\sin 3U}{1-\cos U+\sqrt{3}\sin U}$$

$$=-\frac{\sin 3U}{2\sin^2\frac{U}{2}+\sqrt{3}.2\sin\frac{U}{2}.\cos\frac{U}{2}}$$

$$=-\frac{\sin 3u}{2\sin\frac{U}{2}\left(\sin\frac{U}{2}+\sqrt{3}\cos\frac{U}{2}\right)}$$

$$=-\left[\frac{\sin 3u}{3u}.\frac{6\frac{u}{2}}{2\sin\frac{u}{2}}.\frac{1}{\left(\sin\frac{u}{2}+\sqrt{3}.\cos\frac{u}{2}\right)}\right]$$

$$\lim_{x\to\pi/3}\frac{\sin 3x}{1-2\cos x}=-\lim_{u\to 0}\left[\frac{\sin 3u}{3u}.\frac{6\frac{u}{2}}{2\sin\frac{u}{2}}.\frac{1}{\left(\sin\frac{u}{2}+\sqrt{3}\cos\frac{u}{2}\right)}\right]$$

$$=-3\lim_{u\to 0}\frac{1}{\sin\frac{u}{2}+\sqrt{3}\cos\frac{u}{2}}=-3.\frac{1}{\sqrt{3}}=-\sqrt{3}$$

$$\boxed{\lim_{x\to\pi/3}\frac{\sin 3x}{1-2\cos x}=-\sqrt{3}}$$

115. គណនា $\lim\limits_{x\to a}(a^2-x^2)\operatorname{tg}\dfrac{\pi x}{2a}$

តាង $U=a-x\Rightarrow x=a-U$; $x\to a$ នោះ: $U\to 0$

$$(a^2-x^2)\operatorname{tg}\frac{\pi x}{2a}=(a-x)(a+x)\operatorname{tg}\frac{\pi x}{2a}=U(2a-U)\operatorname{tg}\left(\frac{\pi}{2}-\frac{\pi U}{2a}\right)$$

$$\lim_{x\to a}(a^2-x^2)\operatorname{tg}\frac{\pi x}{2a}=\lim_{U\to 0}U(2a-U)\operatorname{tg}\left(\frac{\pi}{2}-\frac{\pi U}{2a}\right)$$

$$=\lim_{U\to 0}U(2a-U)\frac{1}{\operatorname{tg}\frac{\pi U}{2a}}$$

$$=\lim_{U\to 0}\frac{2a}{\pi}(2a-U)\frac{\frac{\pi U}{2a}}{\operatorname{tg}\frac{\pi U}{2a}}=\frac{4a^2}{\pi}$$

$$\boxed{\lim_{x\to a}(a^2-x^2)\operatorname{tg}\frac{\pi x}{2a}=\frac{4a^2}{\pi}}$$

116. គណនា $\lim\limits_{x\to 0}\dfrac{2x-\sin x}{\sqrt{1-\cos x}}$

$$\frac{2x-\sin x}{\sqrt{1-\cos x}}=\frac{2x-\sin x}{\sqrt{2}\left|\sin\frac{x}{2}\right|}=\frac{2(x-\sin\frac{x}{2}\cos\frac{x}{2})}{\sqrt{2}\left|\sin\frac{x}{2}\right|}$$

$$=2\sqrt{2}\left[\frac{\frac{x}{2}}{\left|\sin\frac{x}{2}\right|}-\frac{1}{2}\frac{\sin\frac{x}{2}}{\left|\sin\frac{x}{2}\right|}\cdot\cos\frac{x}{2}\right]$$

$$=2\sqrt{2}\frac{\frac{x}{2}}{\left|\sin\frac{x}{2}\right|}-\sqrt{2}\frac{\sin\frac{x}{2}}{\left|\sin\frac{x}{2}\right|}\cdot\cos\frac{x}{2}$$

$$\lim_{x\to 0}\frac{2x-\sin x}{\sqrt{1-\cos x}}=\lim_{x\to 0}\left[2\sqrt{2}\frac{\frac{x}{2}}{\left|\sin\frac{x}{2}\right|}-\sqrt{2}\frac{\sin\frac{x}{2}}{\left|\sin\frac{x}{2}\right|}\cdot\cos\frac{x}{2}\right](1)$$

បើ $x\to 0^+$: $\sin\frac{x}{2}>0\Rightarrow\left|\sin\frac{x}{2}\right|=\sin\frac{x}{2}$

$$(1)=\lim_{x\to 0}\left[2\sqrt{2}\frac{\frac{x}{2}}{\sin\frac{x}{2}}-\sqrt{2}\frac{\sin\frac{x}{2}}{\sin\frac{x}{2}}\cdot\cos\frac{x}{2}\right]=2\sqrt{2}-\sqrt{2}=\sqrt{2}$$

បើ $x\to 0^-$: $\sin\frac{x}{2}<0\Rightarrow\left|\sin\frac{x}{2}\right|=-\sin\frac{x}{2}$

$$(1)=\lim_{x\to 0}\left[2\sqrt{2}\frac{\frac{x}{2}}{-\sin\frac{x}{2}}-\sqrt{2}\frac{\sin\frac{x}{2}}{-\sin\frac{x}{2}}\cdot\cos\frac{x}{2}\right]=-2\sqrt{2}+\sqrt{2}=-\sqrt{2}$$

$$\boxed{\begin{array}{l}\lim\limits_{x\to 0^+}\dfrac{2x-\sin x}{\sqrt{1-\cos x}}=\sqrt{2}\\[2ex]\lim\limits_{x\to 0^-}\dfrac{2x-\sin x}{\sqrt{1-\cos x}}=-\sqrt{2}\end{array}}$$

117. គណនា $\lim\limits_{b\to 0}\dfrac{\sin(x+2b)-2\sin(x+b)+\sin x}{b^2}$

$$\frac{\sin(x+2b)-2\sin(x+b)+\sin x}{b^2}=\frac{2\sin(x+b)\cos b-2\sin(x+b)}{b^2}$$

$$=\frac{-\sin(x+b)(2-2\cos b)}{b^2}$$

$$=\frac{-2\sin(x+b)\cdot 2\sin^2\frac{b}{2}}{b^2}$$

$$=\frac{-4\sin(x+b)\cdot\sin^2 b/2}{b^2}$$

$$\lim_{b\to 0}\frac{\sin(x+2b)-2\sin(x+b)+\sin x}{b^2}=\lim_{b\to 0}\frac{-4\sin(x+b)\sin^2\frac{b}{2}}{b^2}$$

$$=\lim_{b\to 0}\left[(-1)\sin(x+b)\left(\frac{\sin\frac{b}{2}}{\frac{b}{2}}\right)^2\right]$$

$$=(-1).\sin x.1=-\sin x$$

$$\boxed{\lim_{b\to 0}\frac{\sin(x+2b)-2\sin(x+b)+\sin x}{b^2}=-\sin x}$$

118. គណនា $\displaystyle\lim_{y\to \pi/3}\frac{\sin(y-\pi/3)}{4\cos^2 y-1}$

របៀបទី១

តាង $y=\frac{\pi}{3}+t$, ពេល $y\to\frac{\pi}{3}$ នោះ $t\to 0$

$$\frac{\sin(y-\pi/3)}{4\cos^2 y-1}=\frac{\sin t}{4\left[\cos^2(\pi/3+t)-\cos^2\pi/3\right]}=\frac{\sin t}{-4\sin t\sin(t+\frac{2\pi}{3})}$$

$$=\frac{-1}{4\sin\left(t+\frac{2\pi}{3}\right)}$$

$$\lim_{y\to\pi/3}\frac{\sin(y-\frac{\pi}{3})}{4\cos^2 y-1}=\lim_{t\to 0}\frac{-1}{4\sin(t+\frac{2\pi}{3})}=-\frac{\sqrt3}{6}$$

$$\boxed{\lim_{y\to\pi/3}\frac{\sin(y-\pi/3)}{4\cos^2 y-1}=\frac{-\sqrt3}{6}}$$

របៀបទី២

$$\frac{\sin(y-\pi/3)}{4\cos^2 y-1}=\frac{\sin(y-\pi/3)}{2(2\cos y+1)(\cos y-\frac{1}{2})}=\frac{\sin(y-\pi/3)}{2(2\cos y+1)(\cos y-\cos\pi/3)}$$

$$=\frac{2\sin(\frac{y}{2}-\frac{\pi}{6})\cos(\frac{y}{2}-\frac{\pi}{6})}{2(2\cos y+1)\left[-2\sin(\frac{y}{2}+\frac{\pi}{6})\sin(\frac{y}{2}-\frac{\pi}{6})\right]}$$

$$=\frac{\cos(\frac{y}{2}-\frac{\pi}{6})}{-2(2\cos y+1).\sin(\frac{y}{2}+\frac{\pi}{6})}$$

$$\lim_{y\to\pi/3}\frac{\sin(y-\pi/3)}{4\cos^2 y-1}=\lim_{y\to\pi/3}\frac{\cos(\frac{y}{2}-\frac{\pi}{6})}{-2(2\cos y+1).\sin(\frac{y}{2}+\frac{\pi}{6})}=\frac{1}{-2.2\frac{\sqrt3}{2}}=\frac{-\sqrt3}{6}$$

$$\boxed{\lim_{y\to\pi/3}\frac{\sin(y-\frac{\pi}{3})}{4\cos^2 y-1}=-\frac{\sqrt3}{6}}$$

119. គណនា $\displaystyle\lim_{x\to 0}\frac{\operatorname{tg}x-\sin x}{\sin^3 x}$

$$\frac{\operatorname{tg}x-\sin x}{\sin^3 x}=\frac{\sin x}{\sin^3 x}\left(\frac{1}{\cos x}-1\right)=\frac{1}{\sin^2 x}\left(\frac{1}{\cos x}-1\right)$$

$$=\frac{1}{\sin^2 x}\left(\frac{1-\cos x}{\cos x}\right)=\frac{2\sin^2\frac{x}{2}}{\sin^2 x\cos x}$$

$$\lim_{x\to 0}\frac{\operatorname{tg}x-\sin x}{\sin^3 x}=\lim_{x\to 0}\frac{2\sin^2\frac{x}{2}}{\sin x\cos x}=\frac{1}{2}$$

$$\boxed{\lim_{x\to 0}\frac{tg\,x-\sin x}{\sin^3 x}=\frac{1}{2}}$$

120. គណនា $\lim\limits_{z\to 1}(1-z)\,tg\,\frac{\pi z}{2}$

$$(1-z)\,tg\,\frac{\pi z}{2}=(1-z)\frac{\sin\frac{\pi z}{2}}{\cos\frac{\pi z}{2}}=(1-z)\frac{\sin\frac{\pi z}{2}}{\sin\left(\frac{\pi}{2}-\frac{\pi z}{2}\right)}$$

$$=\frac{\sin\frac{\pi z}{2}}{\frac{\pi}{2}\cdot\frac{\sin\left[\frac{\pi}{2}(1-z)\right]}{\frac{\pi}{2}(1-z)}}$$

$$\lim_{z\to 1}(1-z)\,tg\,\frac{\pi z}{2}=\lim_{z\to 1}\frac{\sin\frac{\pi z}{2}}{\frac{\sin\left[\frac{\pi}{2}(1-z)\right]}{\frac{\pi}{2}(1-z)}\cdot\frac{\pi}{2}}=\frac{1}{\frac{\pi}{2}}=\frac{2}{\pi}$$

$$\boxed{\lim_{z\to 1}(1-z)\,tg\,\frac{\pi z}{2}=\frac{2}{\pi}}$$

121. គណនា $\lim\limits_{t\to 0}\frac{tg(a-t)\,tg(a+t)-tg^2a}{t^2}$

$$tg(a-t)\,tg(a+t)-tg^2a=\frac{\sin(a-t)\sin(a+t)}{\cos(a-t)\sin(a+t)}-\frac{\sin^2a}{\cos^2a}$$

$$=\frac{\cos2t-\cos2a}{\cos2t+\cos2a}-\frac{1-\cos2a}{1+\cos2a}$$

$$=\frac{(\cos2t-\cos2a)(1+\cos2a)-(1-\cos2a)(\cos2t+\cos2a)}{\cos(a-t)\cos(a+t)\cos^2a}$$

$$=\frac{2\cos2a\cos2t-2\cos2a}{\cos(a-t)\cos(a+t)\cos^2a}$$

$$=\frac{-2\cos2a\,(1-\cos2t)}{\cos(a-t)\cos(a+t)\cos^2a}$$

$$=\frac{-4\cos2a\,\sin^2t}{\cos(a-t)\cos(a+t)\cos^2a}$$

$$\lim_{t\to 0}\frac{tg(a-t)\,tg(a+t)-tg^2a}{t^2}=\lim_{t\to 0}\frac{-4\cos2a\,\sin^2t}{t^2\cos(a-t)\cos(a+t)\cos^2a}$$

$$=\frac{-4\cos2a}{\cos^4a}$$

$$\boxed{\lim_{t\to 0}\frac{tg(a-t)\,tg(a+t)-tg^2a}{t^2}=\frac{-4\cos2a}{\cos^4a}}$$

122. គណនា $\lim\limits_{x\to 0}(1+\sin x)^{cosec\,x}$

$$cosec\,x=\frac{1}{\sin x}$$

$$\lim_{x\to 0}(1+\sin x)^{cosec\,x}=\lim_{x\to 0}(1+\sin x)^{\frac{1}{\sin x}}$$

ដោយដឹងថា $\lim\limits_{\alpha\to 0}(1+\alpha)^{\frac{1}{\alpha}}=e$

$$\boxed{\lim_{x\to 0}(1+\sin x)^{cosec\,x}=e}$$

លំហាត់ 123

គណនា $\lim\limits_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^{x^2}$

$$\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^{x^2}=\lim_{x\to\pm\infty}\left[\left(1+\frac{1}{x}\right)^{x}\right]^{x}$$

$$=e^{\lim\limits_{x\to\pm\infty}x}\begin{cases}e^{\lim\limits_{x\to+\infty}x}=+\infty\\ e^{\lim\limits_{x\to-\infty}x}=0\end{cases}$$

ដូចនេះ:

$$\boxed{\lim_{x\to\pm\infty}\left(1+\frac{1}{x}\right)^{x^2}=\begin{cases}0 & \text{បើ } x\to-\infty\\ +\infty & \text{បើ } x\to+\infty\end{cases}}$$

124. គណនា $\lim\limits_{x\to 0}\frac{\ln(1+kx)}{x}$

តាមរូបមន្ត $\lim\limits_{x\to 0}\frac{\ln(1+ax)}{x}=a$

យើងបាន $\lim\limits_{x\to 0}\frac{\ln(1+kx)}{x}=k$

$$\boxed{\lim_{x\to 0}\frac{\ln(1+kx)}{x}=k}$$

លំហាត់ 125.

គណនា $\lim\limits_{x\to\infty}x\left[\ln(x+a)-\ln x\right]$

យើងបាន

$$\lim_{x\to\infty}x\left[\ln(x+a)-\ln x\right]=\lim_{x\to\infty}x\ln\left(\frac{x+a}{x}\right)=\lim_{x\to\infty}\ln\left(1+\frac{a}{x}\right)^{x}$$

$$=\ln\left[\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^{\frac{x}{a}\cdot a}\right]=\ln\left[e^{a}\right]=a$$

$$\boxed{\lim_{x\to\infty}x\left[\ln(x+a)-\ln x\right]=a}$$

126. គេឲ្យផលគុណ $P=\cos\frac{x}{2}\cdots\cos\frac{x}{2^{n-1}}\cdot\cos\frac{x}{2^n}$

ដែល $\frac{x}{2^n}\neq k\pi\ (k\in\mathbb{Z})\ (n\in\mathbb{N})$

ក/ ចំពោះ x មិនប្រែប្រួល រក $\lim\limits_{n\to\infty}P$

ខ/ ចំពោះ n មិនប្រែប្រួល រក $\lim\limits_{x\to 0}P$

ដំណោះស្រាយ

ក/ រក $\lim\limits_{n\to\infty}P$

ដោយ $\frac{x}{2^n}\neq k\pi\Rightarrow\sin\frac{x}{2^n}\neq 0$

គុណអង្គទាំងពីរនៃផលគុណ P និង $2^n\sin\frac{x}{2^n}$ យើងបាន

$$2^n\sin\frac{x}{2^n}\cdot P=2\cos\frac{x}{2}\cdot 2\cos\frac{x}{4}\cdots 2\cos\frac{x}{2^{n-1}}\cdot 2\cos\frac{x}{2^n}\cdot\sin\frac{x}{2^n}$$

យើងមានទៀត $\sin 2a = 2\sin a\cos a$ ដោយធ្វើដូចគ្នា បាន

$$2^n \sin\frac{x}{2^n}\cdot P = 2\cos\frac{x}{2}\cdot 2\cos\frac{x}{4}\cdots 2\cos\frac{x}{2^{n-1}}\cdot\sin\frac{x}{2^{n-1}}$$

$$2^n \sin\frac{x}{2^n}\cdot P = 2\cos\frac{x}{2}\,\sin\frac{x}{2} = \sin x$$

$$\Rightarrow P = \frac{\sin x}{2^n \sin\frac{x}{2^n}}$$

$$\lim_{n\to\infty} P = \lim_{n\to\infty}\frac{\sin x}{2^n\sin\frac{x}{2^n}} = \lim_{n\to\infty}\frac{\sin x}{x\cdot\frac{\sin\frac{x}{2^n}}{\frac{x}{2^n}}} = \frac{\sin x}{x}$$

$$\boxed{\lim_{n\to\infty} P = \frac{\sin x}{x}}$$

$$\lim_{x\to 0} P = \lim_{x\to 0}\frac{\frac{\sin x}{x}}{\frac{\sin\frac{x}{2^n}}{\frac{x}{2^n}}} = 1$$

$$\boxed{\lim_{x\to 0} P = 1}$$

127. រកលីមីតរបស់បរិមាត្រ P_n ពហុកោណនិយ័ត n ជ្រុង ចារឹកក្នុងរង្វង់កាំ R ពេលដែលជ្រុង $n\to\infty$, ដូចគ្នានេះដែរ ចំពោះបរិមាត្រ Q_n របស់ពហុកោណនិយ័តចារឹកក្រៅរង្វង់ ។

ដំណោះស្រាយ

* **រកលីមីត P_n**

ពហុកោណនេះជាពហុកោណនិយ័តចារឹកក្នុងរង្វង់កាំ R ដូច្នេះ បរិមាត្រ $P_n = n[AB]$ (ដូចរូបទី1) ។

យើងមានទៀត $|AB| = 2R\sin\frac{\pi}{n}$

ដូច្នេះ $\lim_{n\to\infty} P_n = \lim_{n\to\infty} 2nR\sin\frac{\pi}{n}$

$n\to\infty \Rightarrow \frac{\pi}{n}\to 0$

$$\lim_{n\to\infty}\frac{2nR\sin\frac{\pi}{n}}{\frac{\pi}{n}}\times\frac{\pi}{n} = \lim_{n\to\infty} 2nR\frac{\pi}{n} = 2R\pi$$

$$\boxed{\lim_{n\to\infty} P_n = 2\pi R}$$

* **រកលីមីត Q_n**

ដោយពហុកោណនេះជាពហុកោណចារឹកក្រៅរង្វង់កាំ R

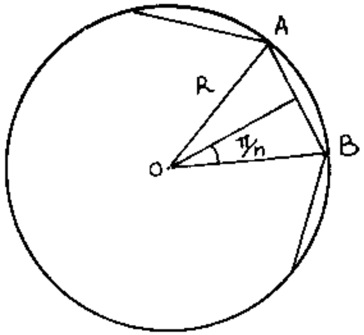
ដូចនេះ: $Q_n = n|AB|$ (ដូចរូបទី2)

ប៉ុន្តែ $|AB| = 2R\,\mathrm{tg}\frac{\pi}{n}$

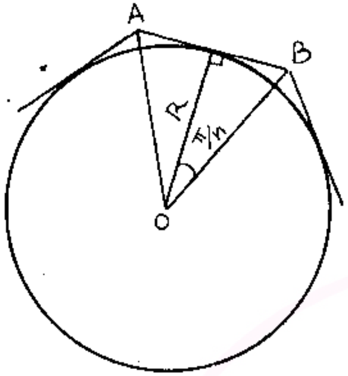
$$\lim_{n\to\infty} Q_n = \lim_{n\to\infty} 2nR\,\mathrm{tg}\frac{\pi}{n} = \lim_{n\to\infty} 2nR\,\frac{\sin\frac{\pi}{n}}{\frac{\pi}{n}}\times\frac{\frac{\pi}{n}}{\cos\frac{\pi}{n}}$$

$$= \lim_{n\to\infty}\frac{2nR\,\frac{\pi}{n}}{\cos\frac{\pi}{n}} = 2\pi R$$

$$\boxed{\lim_{n\to\infty} Q_n = 2\pi R}$$



(រូបទី1)



(រូបទី2)

---

128. ចូរស្រាយបញ្ជាក់ថា $\lim\limits_{n\to-\infty}\left(\dfrac{1^3+2^3+\cdots+n^3}{n^3}-\dfrac{n}{4}\right)=\dfrac{1}{2}$

តាង : $S_3 = 1^3+2^3+\cdots+n^3$

$S_2 = 1^2+2^2+\cdots+n^2$

$S_1 = 1+2+\cdots+n = \dfrac{(n+1).n}{2}$

គណនា $S_2 = 1^2+2^2+\cdots+2^n$

$$1^3 = 1$$
$$(1+1)^3 = 1+3.1+3.1^2+1^3$$
$$(1+2)^3 = 1+3.2+3.2^2+2^3$$
$$\vdots$$
$$[1+(n-1)]^3 = 1+3(n-1)+3(n-1)^2+(n-1)^3$$
$$(1-n)^3 = 1+3n+3n^2+n^3$$

$$(n+1)^3 = (n+1)+3(1+2+\cdots+n)+3(1^2+2^2+\cdots+n^2)$$

$$= (n+1)+3S_1+3S_2$$

$$\Rightarrow 3S_2 = (n+1)^3-(n+1)-3S_1 = (r+1)^3-(n+1)-3\,\frac{n(n+1)}{2}$$

$$= (n+1)\left[(n+1)^2-1-\frac{3n}{2}\right] = (n+1)\left(\frac{2(n^2+2n+1)-2-3n}{2}\right)$$

$$= (n+1)\left(\frac{2n^2+4n+2-2-3n}{2}\right) = \left(\frac{n+1}{2}\right)(2n^2+n)$$

$$= \frac{n}{2}(n+1)(2n+1)$$

$$\Rightarrow S_2 = \frac{n}{6}(n+1)(2n+1)$$

គណនា $S_3 = 1^3+2^3+\cdots+n^3$

$$1^4 = 1.$$
$$(1+1)^4 = 1+4.1+6.1^2+4.1^3+1^4$$
$$(1+2)^4 = 1+4.2+6.2^2+4.2^3+2^4$$
$$[1+(n-1)]^4 = 1+4(n-1)+6(n-2)^2+4(n-2)^3+(n-2)^4$$
$$(1+n)^4 = 1+4n+6n^2+4n^3+n^4$$

$$(n+1)^4 = (n+1)+4(1+2+\cdots+n)+6(1^2+2^2+\cdots+n^2)+4(1^3+2^3+\cdots+n^3)$$

$$= (n+1)+4S_1+6S_2+4S_3$$

$$\Rightarrow 4S_3 = (n+1)^4-4S_1-6S_2-(n+1)$$

$$= (n+1)^4-(n+1)-4\,\frac{n(n+1)}{2}-6\,\frac{n(n+1)(2n+1)}{6}$$

$$= (n+1)^4-(n+1)-2n(n+1)-n(n+1)(2n+1)$$

$$= (n+1)\left[(n+1)^3-1-2n-n(2n+1)\right]$$

$$=(n+1)\left[\left((n+1)^3-1\right)-n(2+2n+1)\right]$$

$$=(n+1)\left\{\left[(n+1-1)\left((n+1)^2-(m+1)+1^2\right]-n(2n+3)\right\}$$

$$=(n+1)\left[n\left(n^2+2n+1+n+2\right)-n(2n+3)\right]$$

$$=(n+1)n\left(n^3+3n+3-2n-3\right)=n(n+1)\left(n^2+n\right)$$

$$=n^2(n+1)^2$$

$$\Rightarrow S_3=\frac{n^2}{4}(n+1)^2$$

តាង $F_{(n)}=\dfrac{1^3+2^3+\dots+n^3}{n^3}-\dfrac{n}{4}$

$$=\frac{n^2(n+1)^2}{4n^3}-\frac{n}{4}=\frac{(n+1)^2}{4n}-\frac{n}{4}=\frac{(n+1)^2-n^2}{4n}$$

$$=\frac{n^2+2n+1-n^2}{4n}=\frac{2n+1}{4n}=\frac{n\left(2+\frac{1}{n}\right)}{4n}=\frac{2+\frac{1}{n}}{4}\quad(n\neq 0)$$

$$\lim_{n\to+\infty}F_{(n)}=\lim_{n\to+\infty}\frac{2+\frac{1}{n}}{4}=\frac{2}{4}=\frac{1}{2}$$

$$\boxed{\lim_{n\to+\infty}F_{(n)}=\frac{1}{2}}$$

129. រកលីមីតនៃឫស x_1 និង x_2 នៃសមីការដឺក្រេទី2
$ax^2+bx+c=0$, កាលណា $a\to 0$ ហើយ b និង c ថេរ $(b\neq 0)$ ។

ដំណោះស្រាយ

ករណីដែលសមីការ $ax^2+bx+c=0$ មានឫសពីរ

$$x_1=\frac{-b-\sqrt{b^2-4ac}}{2a}\ ,\quad x_2=\frac{-b+\sqrt{b^2-4ac}}{2a}$$

$$x_1=\frac{-b-\sqrt{b^2-4ac}}{2a}$$

$\underline{b\neq 0}$: $\sqrt{b^2-4ac}=\sqrt{b^2\left(1-\frac{4ac}{b^2}\right)}=|b|\sqrt{1-\frac{4ac}{b^2}}$

$\underline{b>0}$: $|b|=b$; $\sqrt{b^2-4ac}=b\cdot\sqrt{1-\frac{4ac}{b^2}}$

$$+\ x_1=\frac{-b-\sqrt{b^2-4ac}}{2a}=\frac{-b-b\sqrt{1-\frac{4ac}{b^2}}}{2a}=\frac{-b\left(1+\sqrt{1-\frac{4ac}{b^2}}\right)}{2a}$$

$$\lim_{a\to 0}x_1=-b\lim_{a\to 0}\frac{1+\sqrt{1-\frac{4ac}{b^2}}}{2a}=\frac{-2b}{2\lim\limits_{a\to 0}a}=\frac{-b}{0}=\underline{\infty}$$

$$\lim_{a\to 0^+}x_1=\frac{-b}{\lim\limits_{a\to 0^+}a}=\underline{-\infty}$$

$$\lim_{a\to 0^-}x_1=\frac{-b}{\lim\limits_{a\to 0}a}=\underline{+\infty}$$

$\underline{b<0}$: $|b|=-b$; $b^2-4ac=-b\sqrt{1-\frac{4ac}{b^2}}$

$$x_1=\frac{-b-\sqrt{b^2-4ac}}{2a}=\frac{-b+b\sqrt{1-\frac{4ac}{b^2}}}{2a}=\frac{-b\left(1-\sqrt{1-\frac{4ac}{b^2}}\right)}{2a}$$

$\lim\limits_{a\to 0} x_1$ មានរាងមិនកំណត់ $\frac{0}{0}$

$$x_1=\frac{-b-\sqrt{b^2-4ae}}{2a}=\frac{\left(-b-\sqrt{b^2-4ae}\right)\left(-b+\sqrt{b^2-4ae}\right)}{2a\left(-b+\sqrt{b^2-4ae}\right)}$$

$$=\frac{b^2-(b^2-4ae)}{2a\left(-b-b\sqrt{1-\frac{4ae}{b^2}}\right)}=\frac{4ae}{-2ab\left(1+\sqrt{1-\frac{4ae}{b^2}}\right)}$$

$$=\frac{-2e}{b\left(1+\sqrt{1-\frac{4ae}{b^2}}\right)}$$

$$\lim_{a\to 0} x_1=\lim_{a\to 0}\frac{-2e}{b}\cdot\frac{1}{1+\sqrt{1-\frac{4ae}{b^2}}}=\frac{-2e}{2b}=\underline{-\frac{e}{b}} \text{ ពិត}$$

$$+\ x_2=\frac{-b+\sqrt{b^2-4ae}}{2a}$$

$\underline{b\neq 0}$: $\sqrt{b^2-4ae}=\sqrt{b^2\left(1-\frac{4ae}{b^2}\right)}=|b|\sqrt{1-\frac{4ae}{b^2}}$

$\underline{b>0}$: $|b|=b$; $\sqrt{b^2-4ae}=b\sqrt{1-\frac{4ae}{b^2}}$

$$x_2=\frac{-b+\sqrt{b^2-4ae}}{2a}=\frac{-b+b\sqrt{1-\frac{4ae}{b^2}}}{2a}=\frac{-b\left(1-\sqrt{1-\frac{4ae}{b^2}}\right)}{2a}$$

$\lim\limits_{a\to 0} x_2$ មានរាងមិនកំណត់ $\frac{0}{0}$

$$x_2=\frac{-b+\sqrt{b^2-4a}}{2a}=\frac{\left(-b+\sqrt{b^2-4ae}\right)\left(-b-\ b^2-4ae\right.}{2a\left(-b-\sqrt{b^2-4ae}\right)}$$

$$=\frac{b^2-(b^2-4ae)}{2a\left(-b-b\sqrt{1-\frac{4ae}{b^2}}\right)}=\frac{4ae}{-2ab\left(1+\sqrt{1-\frac{4ae}{b^2}}\right)}=\frac{-2e}{b\left(1+\sqrt{1-\frac{4ae}{b^2}}\right)}$$

$$\lim_{a\to 0} x_2=\lim_{a\to 0}\frac{-2e}{b\left(1+\sqrt{1-\frac{4ae}{b^2}}\right)}=\underline{-\frac{e}{b}} \text{ ពិត}$$

$\underline{b<0}$: $|b|=-b$; $\sqrt{b^2-4ae}=-b\sqrt{1-\frac{4ae}{b^2}}$

$$x_2=\frac{-b+\sqrt{b^2-4ae}}{2a}=\frac{-b-b\sqrt{1-\frac{4ae}{b^2}}}{2a}=\frac{-b\left(1+\sqrt{1-\frac{4ae}{b^2}}\right)}{2a}$$

$$\lim_{a\to 0} x_2=\lim_{a\to 0}\frac{-b\left(1+\sqrt{1-\frac{4ae}{b^2}}\right)}{2a}=\frac{-2b}{2\lim\limits_{a\to 0} a}=\frac{-b}{0}=\underline{\infty}$$

$$\lim_{a\to 0^+} x_2=\frac{-b}{\lim\limits_{a\to 0} a}=\underline{+\infty}$$

$$\lim_{a\to 0} x_2=\frac{-b}{\lim\limits_{a\to 0} a}=\underline{-\infty}$$

<u>ករណីពិសេស</u> បើ $x_1=x_2=\frac{-b}{2a}$

$$\lim_{a\to 0} x_1=\frac{-b}{2\lim\limits_{a\to 0} a}=\underline{\infty}$$

$\underline{b>0}$: $\lim\limits_{a\to 0^+} x_1=\frac{-b}{2\lim\limits_{a\to 0^+} a}=\underline{-\infty}$

$$\lim_{a \to 0^-} x_1 = \frac{-b}{2 \lim\limits_{a \to 0^-} a} = +\infty$$

$\underline{b<0}$ $$\lim_{a \to 0^+} x_1 = \frac{-b}{2 \lim\limits_{a \to 0} a} = +\infty$$

$$\lim_{a \to 0^-} x_1 = \frac{-b}{2 \lim\limits_{a \to 0} a} = -\infty$$

- ចប់ -